

Question 1:

Evaluate:

$$(i) 3^{-2} \qquad (ii) (-4)^{-2} \qquad (iii) \left(\frac{1}{2}\right)^{-5}$$

Answer 1:

$$(i) \quad 3^{-2} = \frac{1}{3^2} \qquad \left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$= \frac{1}{9}$$

$$(ii) \quad (-4)^{-2} = \frac{1}{(-4)^2} \qquad \left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$= \frac{1}{16}$$

$$(iii) \quad \left(\frac{1}{2}\right)^{-5} = \left(\frac{2}{1}\right)^5 \qquad \left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$= (2)^5 = 32$$

Question 2:

Simplify and express the result in power notation with positive exponent:

$$(i) (-4)^5 \div (-4)^8 \qquad (ii) \left(\frac{1}{2^3}\right)^2 \qquad (iii) (-3)^4 \times \left(\frac{5}{3}\right)^4$$

$$(iv) (3^{-7} \div 3^{-10}) \times 3^{-5} \qquad (v) 2^{-3} \times (-7)^{-3}$$

Answer 2:

$$(i) \quad (-4)^5 \div (-4)^8 = (-4)^{5-8} \qquad \left[\because a^m \div a^n = a^{m-n} \right]$$

$$= (-4)^{-3} = \frac{1}{(-4)^3} \qquad \left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$(ii) \quad \left(\frac{1}{2^3}\right)^2 = \frac{1^2}{(2^3)^2} \qquad \left[\because \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \right]$$

$$= \frac{1}{2^{3 \times 2}} = \frac{1}{2^6} \qquad \left[\because (a^m)^n = a^{m \times n} \right]$$

$$(iii) \quad (-3)^4 \times \left(\frac{5}{3}\right)^4 = (-3)^4 \times \frac{5^4}{3^4} \qquad \left[\because \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \right]$$

$$= \{(-1)^4 \times 3^4\} \times \frac{5^4}{3^4} \qquad \left[\because (ab)^m = a^m b^m \right]$$

$$= 3^{4-4} \times 5^4 \qquad \left[\because a^m \div a^n = a^{m-n} \right]$$

$$= 3^0 \times 5^4 = 5^4 \qquad \left[\because a^0 = 1 \right]$$

$$(iv) \quad (3^{-7} \div 3^{-10}) \times 3^{-5} = 3^{-7-(-10)} \times 3^{-5} \qquad \left[\because a^m \div a^n = a^{m-n} \right]$$

$$= 3^{-7+10} \times 3^{-5} = 3^3 \times 3^{-5} = 3^{3+(-5)} \qquad \left[\because a^m \times a^n = a^{m+n} \right]$$

$$(v) \quad 2^{-3} \times (-7)^{-3} = \frac{1}{2^3} \times \frac{1}{(-7)^3} \quad \left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$= \frac{1}{\{2 \times (-7)\}^3} = \frac{1}{(-14)^3} \quad \left[\because (ab)^m = a^m b^m \right]$$

Question 3:

Find the value of:

$$(i) \quad (3^0 + 4^{-1}) \times 2^2 \quad (ii) \quad (2^{-1} \times 4^{-1}) \div 2^{-2} \quad (iii) \quad \left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$$

$$(iv) \quad (3^{-1} + 4^{-1} + 5^{-1})^0 \quad (v) \quad \left\{ \left(\frac{-2}{3}\right)^{-2} \right\}^2$$

Answer 3:

$$(i) \quad (3^0 + 4^{-1}) \times 2^2 = \left(1 + \frac{1}{4}\right) \times 2^2 \quad \left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$= \left(\frac{4+1}{4}\right) \times 2^2 = \frac{5}{4} \times 2^2 = \frac{5}{2^2} \times 2^2$$

$$= 5 \times 2^{2-2} \quad \left[\because a^m \div a^n = a^{m-n} \right]$$

$$= 5 \times 2^0 = 5 \times 1 = 5 \quad \left[\because a^0 = 1 \right]$$

$$(ii) \quad (2^{-1} \times 4^{-1}) \div 2^{-2} = \left(\frac{1}{2^1} \times \frac{1}{4^1}\right) \div 2^{-2} \quad \left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$= \left(\frac{1}{2} \times \frac{1}{2^2}\right) \div 2^{-2} = \frac{1}{2^3} \div 2^{-2} \quad \left[\because a^m \times a^n = a^{m+n} \right]$$

$$= 2^{-3} \div 2^{-2} = 2^{-3-(-2)} = 2^{-3+2} = 2^{-1} \quad \left[\because a^m \div a^n = a^{m-n} \right]$$

$$= \frac{1}{2} \quad \left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$(iii) \quad \left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2} = (2^{-1})^{-2} + (3^{-1})^{-2} + (4^{-1})^{-2} \quad \left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$= 2^{-1 \times (-2)} + 3^{-1 \times (-2)} + 4^{-1 \times (-2)} \quad \left[\because (a^m)^n = a^{m \times n} \right]$$

$$= 2^2 + 3^2 + 4^2 = 4 + 9 + 16 = 29$$

$$(iv) \quad (3^{-1} + 4^{-1} + 5^{-1})^0 = \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{5}\right)^0 \quad \left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$= \left(\frac{20+15+12}{60}\right)^0 = \left(\frac{47}{60}\right)^0 = 1 \quad \left[\because a^0 = 1 \right]$$

$$(v) \quad \left\{ \left(\frac{-2}{3}\right)^{-2} \right\}^2 = \left(\frac{-2}{3}\right)^{-2 \times 2} \quad \left[\because (a^m)^n = a^{m \times n} \right]$$

$$= \left(\frac{-2}{3}\right)^{-4} = \left(\frac{-3}{2}\right)^4$$

$$= \frac{81}{16}$$

$$\left[\because a^{-m} = \frac{1}{a^m} \right]$$

Question 4:

Evaluate:

$$(i) \frac{8^{-1} \times 5^3}{2^{-4}}$$

$$(ii) (5^{-1} \times 2^{-1}) \times 6^{-1}$$

Answer 4:

$$(i) \frac{8^{-1} \times 5^3}{2^{-4}} = \frac{(2^3)^{-1} \times 5^3}{2^{-4}} = \frac{2^{-3} \times 5^3}{2^{-4}}$$

$$= 2^{-3-(-4)} \times 5^3 = 2^{-3+4} \times 5^3$$

$$= 2 \times 125 = 250$$

$$\left[\because (a^m)^n = a^{m \times n} \right]$$

$$\left[\because a^m \div a^n = a^{m-n} \right]$$

$$(ii) (5^{-1} \times 2^{-1}) \times 6^{-1} = \left(\frac{1}{5} \times \frac{1}{2}\right) \times \frac{1}{6}$$

$$= \frac{1}{10} \times \frac{1}{6} = \frac{1}{60}$$

$$\left[\because a^{-m} = \frac{1}{a^m} \right]$$

Question 5:

Find the value of m for which $5^m \div 5^{-3} = 5^5$.

Answer 5:

$$5^m \div 5^{-3} = 5^5$$

$$\Rightarrow 5^{m-(-3)} = 5^5$$

$$\Rightarrow 5^{m+3} = 5^5$$

$$\left[\because a^m \div a^n = a^{m-n} \right]$$

Comparing exponents both sides, we get

$$\Rightarrow m+3=5$$

$$\Rightarrow m=5-3$$

$$\Rightarrow m=2$$

Question 6:

Evaluate:

$$(i) \left\{ \left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1} \right\}^{-1}$$

$$(ii) \left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4}$$

Answer 6:

$$(i) \quad \left\{ \left(\frac{1}{3} \right)^{-1} - \left(\frac{1}{4} \right)^{-1} \right\} = \left\{ \left(\frac{3}{1} \right)^1 - \left(\frac{4}{1} \right)^1 \right\} \quad \left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$= \{3 - 4\} = -1$$

$$(ii) \quad \left(\frac{5}{8} \right)^{-7} \times \left(\frac{8}{5} \right)^{-4} = \frac{5^{-7}}{8^{-7}} \times \frac{8^{-4}}{5^{-4}} \quad \left[\because \left(\frac{a}{b} \right)^m = \frac{a^m}{b^m} \right]$$

$$= 5^{-7-(-4)} \times 8^{-4-(-7)} \quad \left[\because a^m \div a^n = a^{m-n} \right]$$

$$= 5^{-7+4} \times 8^{-4+7} = 5^{-3} \times 8^3 = \frac{8^3}{5^3} \quad \left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$= \frac{512}{125}$$

Question 7:

Simplify:

$$(i) \quad \frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}} \quad (t \neq 0)$$

$$(ii) \quad \frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$$

Answer 7:

$$(i) \quad \frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}} = \frac{5^2 \times t^{-4}}{5^{-3} \times 5 \times 2 \times t^{-8}} = \frac{5^{2-(-3)-1} \times t^{-4-(-8)}}{2} \quad \left[\because a^m \div a^n = a^{m-n} \right]$$

$$= \frac{5^{2+3-1} \times t^{-4+8}}{2} = \frac{5^4 \times t^4}{2} = \frac{625}{2} t^4$$

$$(ii) \quad \frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}} = \frac{3^{-5} \times (2 \times 5)^{-5} \times 5^3}{5^{-7} \times (2 \times 3)^{-5}} = \frac{3^{-5} \times 2^{-5} \times 5^{-5} \times 5^3}{5^{-7} \times 2^{-5} \times 3^{-5}} \quad \left[\because (ab)^m = a^m b^m \right]$$

$$= \frac{3^{-5} \times 2^{-5} \times 5^{-5+3}}{5^{-7} \times 2^{-5} \times 3^{-5}} = \frac{3^{-5} \times 2^{-5} \times 5^{-2}}{5^{-7} \times 2^{-5} \times 3^{-5}} \quad \left[\because a^m \times a^n = a^{m+n} \right]$$

$$= 3^{-5-(-5)} \times 2^{-5-(-5)} \times 5^{-2-(-7)} \quad \left[\because a^m \div a^n = a^{m-n} \right]$$

$$= 3^{-5+5} \times 2^{-5+5} \times 5^{-2+7} = 3^0 \times 2^0 \times 5^5$$

$$= 1 \times 1 \times 3125 \quad \left[\because a^0 = 1 \right]$$

$$= 3125$$

Exercise 12.2

Question 1:

Express the following numbers in standard form:

(i) 0.00000000000085

(ii) 0.000000000000942

(iii) 6020000000000000

(iv) 0.00000000837

(v) 31860000000

Answer 1:

(i) $0.00000000000085 = 0.00000000000085 \times \frac{10^{12}}{10^{12}} = 8.5 \times 10^{-12}$

(ii) $0.000000000000942 = 0.000000000000942 \times \frac{10^{12}}{10^{12}} = 9.42 \times 10^{-12}$

(iii) $6020000000000000 = 6020000000000000 \times \frac{10^{15}}{10^{15}} = 6.02 \times 10^{15}$

(iv) $0.00000000837 = 0.00000000837 \times \frac{10^9}{10^9} = 8.37 \times 10^{-9}$

(v) $31860000000 = 31860000000 \times \frac{10^{10}}{10^{10}} = 3.186 \times 10^{10}$

Question 2:

Express the following numbers in usual form:

(i) 3.02×10^{-6}

(ii) 4.5×10^4

(iii) 3×10^{-8}

(iv) 1.0001×10^9

(v) 5.8×10^{12}

(vi) 3.61492×10^6

Answer 2:

(i) $3.02 \times 10^{-6} = \frac{3.02}{10^6} = 0.00000302$

(ii) $4.5 \times 10^4 = 4.5 \times 10000 = 45000$

(iii) $3 \times 10^{-8} = \frac{3}{10^8} = 0.00000003$

(iv) $1.0001 \times 10^9 = 1000100000$

(v) $5.8 \times 10^{12} = 5.8 \times 1000000000000 = 5800000000000$

(vi) $3.61492 \times 10^6 = 3.61492 \times 1000000 = 3614920$

Question 3:

Express the number appearing in the following statements in standard form:

- (i) 1 micron is equal to $\frac{1}{1000000}$ m.
- (ii) Charge of an electron is 0.000,000,000,000,000,000,16 coulomb.
- (iii) Size of a bacteria is 0.0000005 m.
- (iv) Size of a plant cell is 0.00001275 m.
- (v) Thickness of a thick paper is 0.07 mm.

Answer 3:

- (i) 1 micron = $\frac{1}{1000000} = \frac{1}{10^6} = 1 \times 10^{-6}$ m
- (ii) Charge of an electron is 0.000000000000000000016 coulombs.
 $= 0.000000000000000000016 \times \frac{10^{19}}{10^{19}} = 1.6 \times 10^{-19}$ coulomb
- (iii) Size of bacteria = $0.0000005 = \frac{5}{10000000} = \frac{5}{10^7} = 5 \times 10^{-7}$ m
- (iv) Size of a plant cell is $0.00001275 \text{ m} = 0.00001275 \times \frac{10^5}{10^5} = 1.275 \times 10^{-5}$ m
- (v) Thickness of a thick paper = $0.07 \text{ mm} = \frac{7}{100} \text{ mm} = \frac{7}{10^2} = 7 \times 10^{-2}$ mm

Question 4:

In a stack there are 5 books each of thickness 20 mm and 5 paper sheets each of thickness 0.016 mm. What is the total thickness of the stack?

Answer 4:

- Thickness of one book = 20 mm
- Thickness of 5 books = $20 \times 5 = 100$ mm
- Thickness of one paper = 0.016 mm
- Thickness of 5 papers = $0.016 \times 5 = 0.08$ mm
- Total thickness of a stack = $100 + 0.08$
 $= 100.08 \text{ mm} = 100.08 \times \frac{10^2}{10^2} = 1.0008 \times 10^2$ mm