Chapter 16 - Playing With Numbers
Exercise 16.2

Question 1:

If $21y5$ is a multiple of 9, where y is a digit, what is the value of y?

Answer 1:

Since $21y5$ is a multiple of 9.

Therefore according to the divisibility rule of 9, the sum of all the digits should be a multiple of 9.

$\therefore \qquad 2+1+y+5 = 8+y$

$\Rightarrow \qquad 8+y = 9$

$\Rightarrow \qquad y = 1$

Question 2:

If $31z5$ is a multiple of 9, where z is a digit, what is the value of z?

You will find that there are *two* answers for the last problem. Why is this so?

Answer 2:

Since $31z5$ is a multiple of 9.

Therefore according to the divisibility rule of 9, the sum of all the digits should be a multiple of 9.

$\therefore \qquad 3+1+z+5 = 9+z$

$\Rightarrow \qquad 9+z = 9$

$\Rightarrow \qquad z = 0$

If $\qquad 3+1+z+5 = 9+z$

$\Rightarrow \qquad 9+z = 18$

$\Rightarrow \qquad z = 9$

Hence, 0 and 9 are two possible answers.

Question 3:

If 24x is a multiple of 3, where x is a digit, what is the value of x?

(Since 24x is a multiple of 3, its sum of digits 6 + x is a multiple of 3; so 6 + x is one of these numbers: 0, 3, 6, 9, 12, 15, 18 ... But since x is a digit, it can only be that 6 + x = 6 or 9 or 12 or 15. Therefore, x = 0 or 3 or 6 or 9. Thus, x can have any of four different values.)

Answer 3:

Since $24x$ is a multiple of 3.

Therefore according to the divisibility rule of 3, the sum of all the digits should be a multiple of 3.

$$\therefore \qquad 2+4+x=6+x$$

Since x is a digit.

$$\Rightarrow \qquad 6+x=6 \qquad \Rightarrow \qquad x=0$$
$$\Rightarrow \qquad 6+x=9 \qquad \Rightarrow \qquad x=3$$
$$\Rightarrow \qquad 6+x=12 \qquad \Rightarrow \qquad x=6$$
$$\Rightarrow \qquad 6+x=15 \qquad \Rightarrow \qquad x=9$$

Thus, x can have any of four different values.

Question 4:

If 31z5 is a multiple of 3, where z is a digit, what might be the values of z?

Answer 4:

Since $31z5$ is a multiple of 3.

Therefore according to the divisibility rule of 3, the sum of all the digits should be a multiple of 3.

Since z is a digit.

$$\therefore \qquad 3+1+z+5=9+z$$
$$\Rightarrow \qquad 9+z=9 \qquad \Rightarrow \qquad z=0$$

If $\qquad 3+1+z+5=9+z$

$$\Rightarrow \qquad 9+z=12 \qquad \Rightarrow \qquad z=3$$

If $\qquad 3+1+z+5=9+z$

$$\Rightarrow \qquad 9+z=15 \qquad \Rightarrow \qquad z=6$$

If $\qquad 3+1+z+5=9+z$

$$\Rightarrow \qquad 9+z=18 \qquad \Rightarrow \qquad z=9$$

Hence, 0, 3, 6 and 9 are four possible answers.