Exercise 10.1

Question 1:

Fill in the blanks:

(i) The centre of a circle lies in __________ of the circle. (exterior/ interior)

(ii) A point, whose distance from the centre of a circle is greater than its radius lies in __________ of the circle. (exterior/ interior)

(iii) The longest chord of a circle is a __________ of the circle.

(iv) An arc is a __________ when its ends are the ends of a diameter.

(v) Segment of a circle is the region between an arc and __________ of the circle.

(vi) A circle divides the plane, on which it lies, in __________ parts.

Answer 1:

(i) The centre of a circle lies in interior of the circle. (exterior/ interior)

(ii) A point, whose distance from the centre of a circle is greater than its radius lies in exterior of the circle. (exterior/ interior)

(iii) The longest chord of a circle is a diameter of the circle.

(iv) An arc is a semi-circle when its ends are the ends of a diameter.

(v) Segment of a circle is the region between an arc and chord of the circle.

(vi) A circle divides the plane, on which it lies, in two parts.

Question 2:

Write True or False: Give reasons for your answers.

(i) Line segment joining the centre to any point on the circle is a radius of the circle.

(ii) A circle has only finite number of equal chords.

(iii) If a circle is divided into three equal arcs, each is a major arc.

(iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle.

(v) Sector is the region between the chord and its corresponding arc.

(vi) A circle is a plane figure.

Answer 2:

(i) Line segment joining the centre to any point on the circle is a radius of the circle. **True**

(ii) A circle has only finite number of equal chords. **False.** Because, there are infinite number of equal chords in a circle.

(iii) If a circle is divided into three equal arcs, each is a major arc. **False**. Because, each arc will make an angle of 120° at the centre. But major arc make angle greater than 180° at the centre.

(iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle. **True**

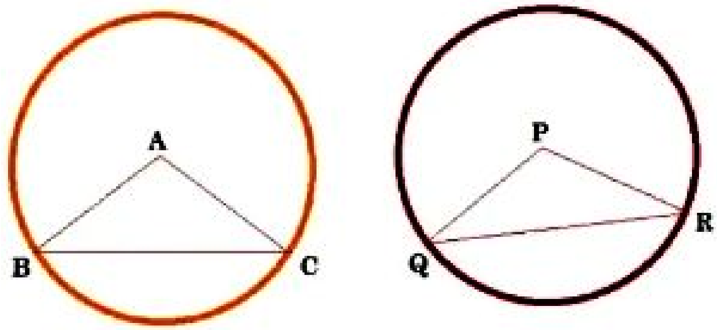
(v) Sector is the region between the chord and its corresponding arc. **False**. Because, between chord and arc a segment is formed. Sector is the region which is formed between radii and arc.

(vi) A circle is a plane figure. **True**

Exercise 10.2

Question 1:
Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.

Answer 1:
Given: Circle C (A, r) and C (P, r) are two congruent circles such that BC = QR.
To prove: ∠BAC = ∠QPR

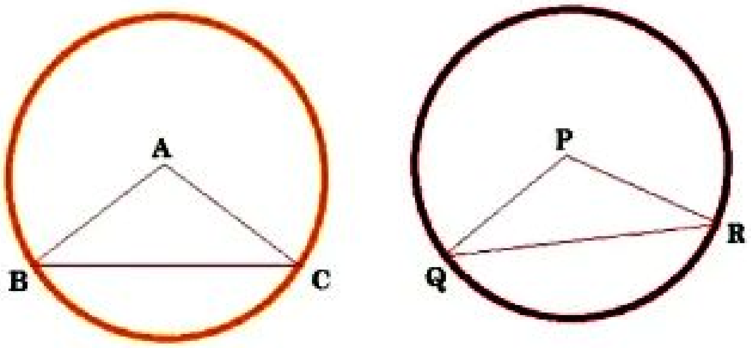


Proof: In ΔABC and ΔPQR,

BC = QR	[∵ Given]
AB = PQ	[∵ Radii of congruent circles]
AC = PR	[∵ Radii of congruent circles]
Hence, ΔABC ≅ ΔPQR	[∵ SSS Congruency rule]
∠BAC = ∠QPR	[∵ CPCT]

Question 2:
Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

Answer 2:
Given: Circle C (A, r) and C (P, r) are two congruent circles such that ∠BAC = ∠QPR.
To prove: BC = QR



Proof: In ΔABC and ΔPQR,

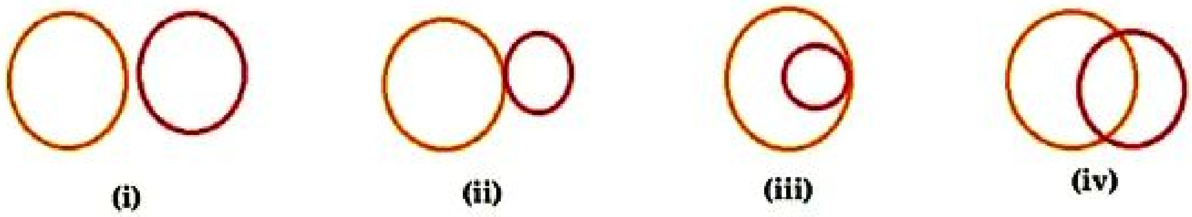
AB = PQ	[∵ Radii of congruent circles]
∠BAC = ∠QPR	[∵ Given]
AC = PR	[∵ Radii of congruent circles]
Hence, ΔABC ≅ ΔPQR	[∵ SSS Congruency rule]
BC = QR	[∵ CPCT]

Exercise 10.3

Question 1:

Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?

Answer 1:

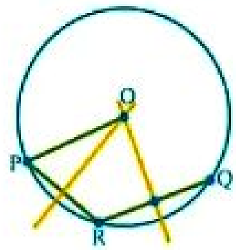


In each pair either 0 or 1 or 2 points are common. The maximum number of common points is 2.

Question 2:

Suppose you are given a circle. Give a construction to find its centre.

Answer 2:



Given: Points P, Q and R lies on circle C (O, r).

Construction:

- Join PR and QR.
- Draw the perpendicular bisectors of PR and QR which intersects at point O.
- Taking O as centre and OP as radius, draw a circle.
- This is the required circle.

Question 3:

If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.

Answer 3:

Given: Circle C (P, r) and circle C (Q, r′) intersects each other at the points A and B.

To prove: Points P and Q lies on the perpendicular bisector of common chord AB.

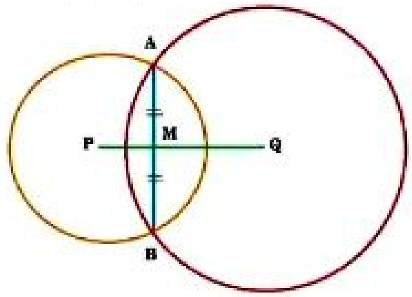
Construction: Join point P and Q to mid-point M of chord AB.

Proof: AB is chord of circle C (P, r) and PM is bisector of chord AB.

Therefore, $PM \perp AB$

[∵ The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.]

Hence, $\angle PMA = 90°$



Similarly, AB is chord of circle C (Q, r′) and QM is bisector of chord AB.

Therefore, $QM \perp AB$

[∵The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.]

Hence, $\angle QMA = 90°$

Now, $\angle PMA + \angle QMA = 90° + 90° = 180°$

Since, $\angle PMA$ and $\angle QMA$ are forming linear pair. So PMQ is a straight line.

Hence, Points P and Q lies on the perpendicular bisector of common chord AB.

Exercise 10.4

Question 1:

Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

Answer 1:

Given: Circle C (P, 3) and circle C (Q, 5) are intersecting at points A and B.

Construction: Join PA and QA. Draw PM as bisector of chord AB.

Proof: AB is chord of circle C (P, 3) and PM is bisector of chord AB.

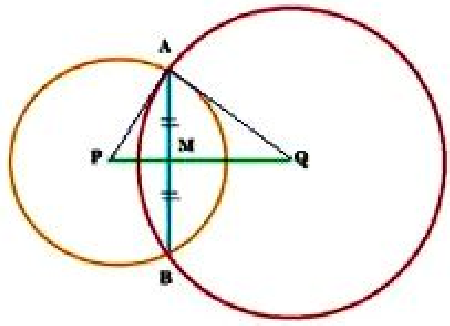
Therefore, PM ⊥ AB

[∵ The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord]

Hence, ∠PMA = 90°

Let, PM = x, therefore, QM = $4 - x$

In ΔAPM, using Pythagoras theorem



$AM^2 = AP^2 - PM^2$ ... (1)

And in ΔAPM, using Pythagoras theorem

$AM^2 = AQ^2 - QM^2$ ... (2)

From the equation (1) and (2), we get

$AP^2 - PM^2 = AQ^2 - QM^2$

$\Rightarrow 3^2 - x^2 = 5^2 - (4 - x)^2 \quad \Rightarrow 9 - x^2 = 25 - (16 + x^2 - 8x)$

$\Rightarrow 9 - 9 = 8x \Rightarrow x = \frac{0}{8} = 0$

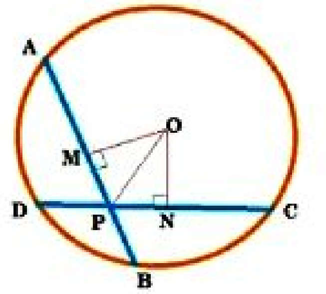
From the equation (1), $AM^2 = 3^2 - 0^2 = 9 \Rightarrow AM = 3$

$\Rightarrow AB = 2AM = 6$

Question 2:

If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Answer 2:



Given: In circle C (O, r), equal chords AB and CD intersects at P.

To prove: AP = CP and BP = DP.

Construction: Join OP. Draw OM ⊥ AB and ON ⊥ CD.

Proof: In ΔOMP and ΔONP,

∠OMP = ∠ONP [∵ Each 90°]

AP = AP [∵ Common]

OM = ON [∵ Equal chords of a circle are equidistant from the centre]

Hence, ΔOMP ≅ ΔONP [∵ RHS Congruency rule]

PM = PN ... (1) [∵ CPCT]

And AB = CD ... (2) [∵ Given]

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}CD$

⇒ AM = CN ... (3)

Adding the equations (1) and (3), we have

AM + PM = CN + PN

⇒ AP = CP ... (4)

Subtracting equation (4) form (2), we have

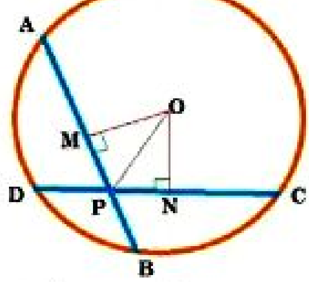
AB – AP = CD – CP

⇒ PB = PD

Question 3:

If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Answer 3:



Given: In circle C (O, r), equal chords AB and CD are intersecting at point P.

To prove: ∠OPM = ∠OPN

Construction: Join OP. Draw OM ⊥ AB and ON ⊥ CD.

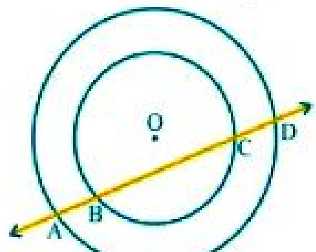
Proof: In ΔOMP and ΔONP,

∠OMP = ∠ONP	[∵ Each 90°]
AP = AP	[∵ Common]
OM = ON	[∵ Equal chords of a circle are equidistant from the centre]
Hence, ΔOMP ≅ ΔONP	[∵ RHS Congruency rule]
∠OPM = ∠OPN	[∵ CPCT]

Question 4:

If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see Figure).

Answer 4:



Given: A line AB is intersecting wo concentric circles with centre O at A, B, C and D.

To prove: AB = CD.

Construction: Draw OM ⊥ AD.

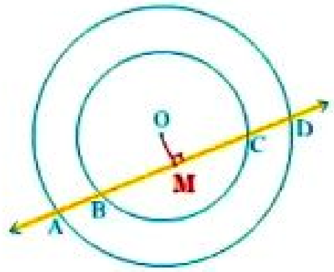
Proof: BC is chord of inner circle and OM ⊥ BC. Therefore

BM = CM ... (1)

[∵ The perpendicular from the centre of a circle to a chord bisects the chord.]

Similarly, AD is chord of outer circle and OM ⊥ AD. Therefore

AM = DM ... (2)



[∵ The perpendicular from the centre of a circle to a chord bisects the chord.]

Subtracting the equation (1) form (2), we get

AM – BM = DM – CM

⇒ AB = CD

Question 5:

Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6m each, what is the distance between Reshma and Mandip?

Answer 5:

Given: In figure, points R, S and M are showing the position of Reshma, Salma and Mandeep respectively. Therefore RS = SM = 6 cm

Construction: Join OR, OS, RS, RM and OM. Draw OL ⊥ RS.

Proof: In ΔORS,

OS = OR and OL ⊥ RS	[∵ By construction]
Therefore, RL = LS = 3 cm	[∵ RS = 6 cm]

In ΔOLS, using Pythagoras theorem, $OL^2 = OS^2 - SL^2$

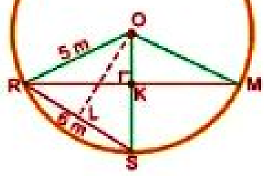
⇒ $OL^2 = 5^2 - 3^2 = 25 - 9 = 16$

⇒ OL = 4

In ΔORK and ΔOMK,

OR = OM [∵ Radii of circle]

∠ROK = ∠MOK [∵ Equal chords subtend equal angle at the centre]
OK = OK [∵ Common]
Hence, ΔORK ≅ ΔOMK [∵ SAS Congruency rule]
RK = MK [∵ CPCT]
Hence, OK ⊥ RM
[∵ The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord]



Now, the area of $\Delta ORS = \frac{1}{2} \times RS \times OL$ ... (1)

And the area of $\Delta ORS = \frac{1}{2} \times OS \times KR$ ... (2)

From the equation (1) and (2),

$$\frac{1}{2} \times RS \times OL = \frac{1}{2} \times OS \times KR$$

$$\Rightarrow RS \times OL = OS \times KR \ \Rightarrow 6 \times 4 = 5 \times KR \ \Rightarrow KR = \frac{6 \times 4}{5} = 4.8$$

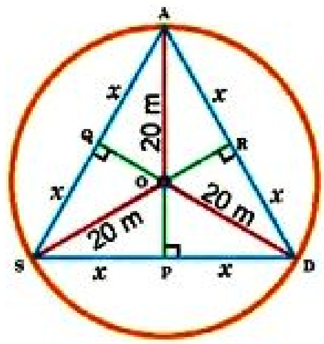
Hence, $RM = 2 \times KR = 2 \times 4.8 = 9.6$ cm

Question 6:

A circular park of radius 20m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Answer 6:

Given: In figure, points A, S and D are the positions of Ankur, Syed and David respectively.
Therefore AS = SD = AD.
Radius of circular park = 20 m, therefore AO = SO = DO = 20
Construction: Draw AP ⊥ SD
Proof: Let AS = SD = AD = $2x$ cm
In ΔASD,
AS = AD and AP ⊥ SD [∵ By construction]
Therefore, SP = PD = x cm [∵ SD = $2x$ cm]



In ΔOPD, using Pythagoras theorem

$$OP^2 = OD^2 - PD^2$$
$$\Rightarrow OP^2 = 20^2 - x^2 = 400 - x^2$$
$$\Rightarrow OP = \sqrt{400 - x^2}$$

Now, in ΔAPD, using Pythagoras theorem

$$AP^2 + PD^2 = AD^2$$
$$\Rightarrow (AO + OP)^2 + x^2 = (2x)^2 \ \Rightarrow \left(20 + \sqrt{400 - x^2}\right)^2 + x^2 = 4x^2$$
$$\Rightarrow 400 + 400 - x^2 + 2 \times 20 \times \sqrt{400 - x^2} + x^2 = 4x^2$$
$$\Rightarrow 800 + 40\sqrt{400 - x^2} = 4x^2 \ \Rightarrow 200 + 10\sqrt{400 - x^2} = x^2$$
$$\Rightarrow 10\sqrt{400 - x^2} = x^2 - 200$$

Squaring both sides

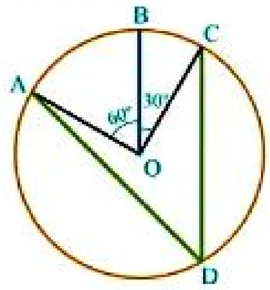
$$100(400 - x^2) = (x^2 - 200)^2$$
$$\Rightarrow 40000 - 100x^2 = x^4 + 40000 - 400x^2$$
$$\Rightarrow x^4 - 300x^2 = 0 \ \Rightarrow x^2(x^2 - 300) = 0$$
$$\Rightarrow x^2 = 300 \ \Rightarrow x = 10\sqrt{3}$$

Hence, the length of the string of each phone $= 2x = 2 \times 10\sqrt{3} = 20\sqrt{3}$ m

Exercise 10.5

Question 1:

In Figure, A, B and C are three points on a circle with centre O such that ∠ BOC = 30° and ∠ AOB = 60°. If D is a point on the circle other than the arc ABC, find ∠ADC.



Answer 1:

∠AOC = ∠AOB + ∠BOC = 60° + 30° = 90°

∠AOC = 2∠ADC

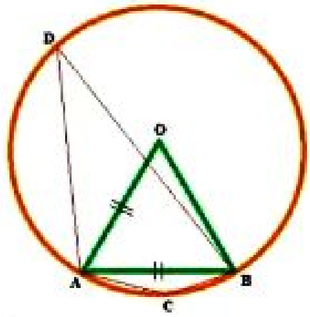
[∵ The angle subtended by an arc at the centre is double the angle subtended by it at any]

⇒∠ADC = $\frac{1}{2}$∠AOC ⇒∠ADC = $\frac{1}{2} \times 90° = 45°$

Question 2:

A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

Answer 2:



Given: In circle C (O, r), OA = AB.

To find: ∠ADB and ∠ACB.

Solution: In ΔOAB

OA = AB [∵ Given]

OA = OB [∵ Radii of circle]

Hence, OA = OB = AB

⇒ ABC is an equilateral triangle.

Therefore, ∠AOB = 60° [∵ Each angle of an equilateral triangle is 60°]

∠AOB = 2∠ADB

[∵ The angle subtended by an arc at the centre is double the angle subtended by it at any]

⇒∠ADB = $\frac{1}{2}$∠AOB ⇒∠ADB = $\frac{1}{2} \times 60° = 30°$

ACBD is a cyclic quadrilateral.

Therefore, ∠ACB + ∠ADB = 180°

[∵ The sum of either pair of opposite angles of a cyclic quadrilateral is 180°.]

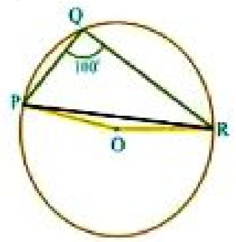
⇒∠ACB + 30° = 180° [∵ ∠ADB = 30°]

⇒∠ACB = 180°−30° = 150°

Question 3:

In Figure, ∠PQR = 100°, where P, Q and R are points on a circle with centre O. Find ∠OPR.

Answer 3:



Construction: Join PS and RS.

PQRS is a cyclic quadrilateral.

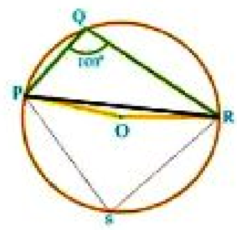
Therefore, ∠PSR + ∠PQR = 180°

[∵The sum of either pair of opposite angles of a cyclic quadrilateral is 180°.]

⇒∠PSR + 100° = 180° [∵ ∠PQR = 100°]

⇒∠PSR = 180°−100° = 80°

Here, ∠POR = 2∠PSR



[∵ The angle subtended by an arc at the centre is double the angle subtended by it at any]

⇒∠POR = 2 × 80° = 160°

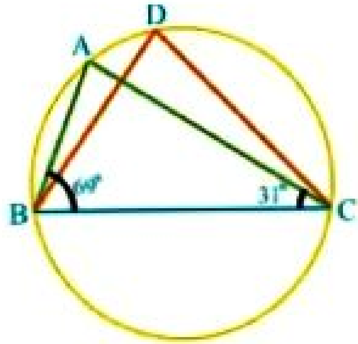
In ΔOPR,

OP = OR [∵ Radii of circle]

$\angle ORP = \angle OPR$ [∵ In an isosceles triangle, the angles opposite to equal sides are equal]
In ΔOPR, $\angle ORP + \angle OPR + \angle POR = 180°$
$\Rightarrow \angle OPR + \angle OPR + 160° = 180°$ [∵$\angle ORP = \angle OPR$]
$\Rightarrow 2\angle OPR + 160° = 180°$
$\Rightarrow 2\angle OPR = 180° - 160° = 20°$
$\Rightarrow \angle OPR = \frac{20°}{2} = 10°$

Question 4:

In Figure, $\angle ABC = 69°$, $\angle ACB = 31°$, find $\angle BDC$.

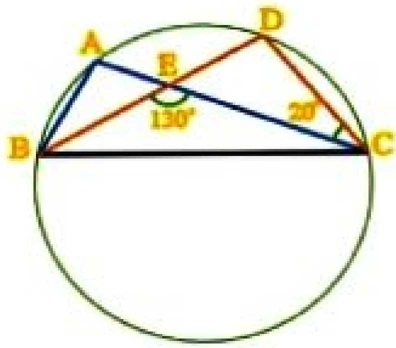


Answer 4:

In ΔABC,
$\angle A + \angle ABC + \angle ACB = 180°$
$\Rightarrow \angle A + 69° + 31° = 180°$ [∵$\angle ABC = 69°$ and $\angle ACB = 31°$]
$\Rightarrow \angle A + 100° = 180°$
$\Rightarrow \angle A = 180° - 100° = 80°$
$\angle BDC = \angle A$ [∵ Angles in the same segments are equal]
$\Rightarrow \angle BDC = 80°$

Question 5:

In Figure, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that $\angle BEC = 130°$ and $\angle ECD = 20°$. Find $\angle BAC$.

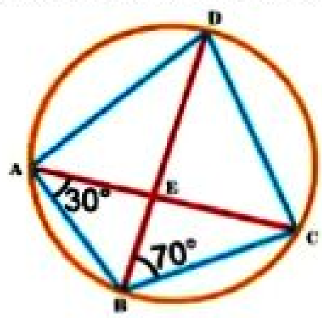


Answer 5:

$\angle DEC + \angle BEC = 180°$ [∵ Linear pair]
$\Rightarrow \angle DEC + 130° = 180°$ [∵$\angle BEC = 130°$]
$\Rightarrow \angle DEC = 180° - 130° = 50°$
In ΔDEC, $\angle D + \angle DEC + \angle DCE = 180°$
$\Rightarrow \angle D + 50° + 20° = 180°$ [∵$\angle DEC = 50°$ and $\angle DCE = 20°$]
$\Rightarrow \angle D + 70° = 180°$
$\Rightarrow \angle D = 180° - 70° = 110°$
$\angle BAC = \angle D$ [∵ Angles in the same segments are equal]
$\Rightarrow \angle BAC = 110°$

Question 6:

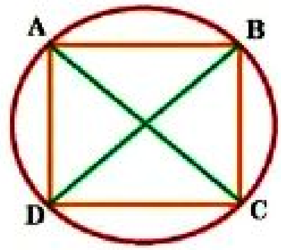
ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70°$, $\angle BAC$ is 30°, find $\angle BCD$. Further, if AB = BC, find $\angle ECD$.



Answer 6:

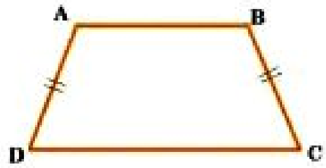
$\angle BDC = \angle BAC$ [∵ Angles in the same segments are equal]
$\Rightarrow \angle BDC = 30°$
In ΔBDC, $\angle BCD + \angle BDC + \angle DBC = 180°$
$\Rightarrow \angle BCD + 30° + 70° = 180°$ [∵$\angle DBC = 70°$ and $\angle BDC = 30°$]
$\Rightarrow \angle BCD + 100° = 180°$ $\Rightarrow \angle BCD = 180° - 100° = 80°$
If AB = BC, then
$\angle BCA = \angle BAC$ [∵In an isosceles triangle, the angles opposite to equal sides are equal]
$\Rightarrow \angle BCA = 30°$
Here $\angle ECD + \angle BCE = \angle BCD$
$\Rightarrow \angle ECD + 30° = 80°$ [∵$\angle BCE = 30°$ and $\angle BCD = 80°$]
$\Rightarrow \angle ECD = 80° - 30° = 50°$

Question 7:
If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

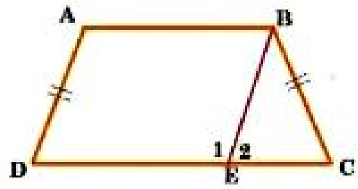
Answer 7:


AC is diameter of circle.
Hence, ∠ADC = 90° and ∠ABC = 90° ... (1) [∵ Angle in a semicircle is a right angle.]
Similarly, BD is diameter of circle.
Hence, ∠BAD = 90° and ∠BCD = 90° ... (2) [∵ Angle in a semicircle is a right angle.]
From the equation (1) and (2), ∠ADC =∠ABC = ∠BAD = ∠BCD = 90°
Hence, ABCD is a rectangle.

Question 8:
If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

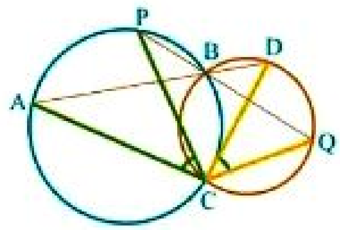
Answer 8:


Given: In trapezium ABCD, AB || DC and AD = BC.
Construction: Draw AD || BE.
Proof: In quadrilateral ABED,



AB || DE [∵ Given]
AD || BE [∵ By construction]
Hence, ABED is a parallelogram.
AD = BE [∵ Opposite sides of a parallelogram are equal]
AD = BC [∵ Given]
⇒ BE = BC
In ΔEBC, BE = BC [∵ Proved above]
Hence, ∠C = ∠2 ... (1) [∵ In an isosceles triangle, the angles opposite to equal sides are equal]
∠A = ∠1 ... (2) [∵ Opposite angles of a parallelogram are equal]
Here, ∠1 + ∠2 = 180° [∵ Linear pair]
⇒∠A + ∠C = 180° [∵ From the equation (1) and (2)]
⇒ ABED is a cyclic quadrilateral.

Question 9:
Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see Figure). Prove that ∠ ACP = ∠ QCD.

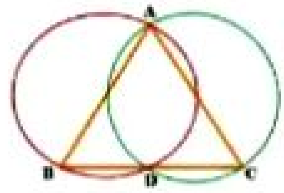
Answer 9:


∠ACP = ∠ABP ... (1) [∵ Angles in the same segments are equal]
∠ABP = ∠QBD ... (2) [∵ Vertically Opposite Angles]
∠QBD = ∠QCD ... (3) [∵ Angles in the same segments are equal]
From the equation (1), (2) and (3),
∠ACP = ∠QCD

Question 10:
If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Answer 10:
Given: Taking AB and AC as diameter two circles are drawn, which intersects each other at D.

Construction: Join AD.

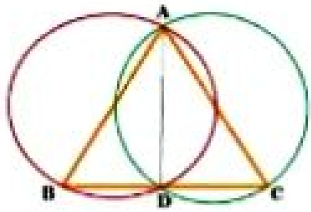


Proof: AB is diameter of circle and ∠ADB is formed in semi-circle.

Hence, ∠ADB = 90° ... (1) [∵ Angle in a semicircle is a right angle]

Similarly, AC is diameter and ∠ADC is formed in semi-circle.

Hence, ∠ADC = 90° ... (2) [∵ Angle in a semicircle is a right angle]



Here, ∠ADB + ∠ADC = 90° + 90° = 180°

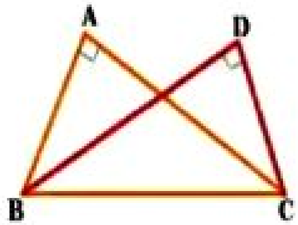
∠ADB and ∠ADC are forming linear pair. Therefore BDC is a straight line.

Hence, the point D lies on third side BC.

Question 11:

ABC and ADC are two right triangles with common hypotenuse AC. Prove that ∠CAD = ∠CBD.

Answer 11:



Given: Triangle ABC and ADC are two right triangle on common base AC.

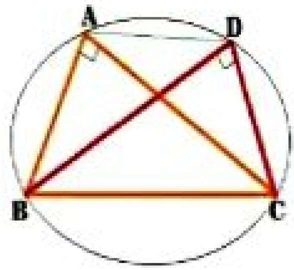
To prove: ∠CAD = ∠CBD.

Proof: Triangle ABC and ADC are on common base BC and ∠BAC = ∠BDC.

Hence, points A, B, C and D lie on the same circle.

[∵ If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle.]

Therefore,



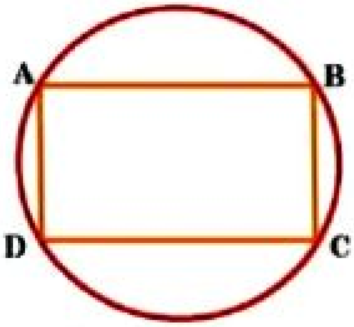
∠CAD = ∠CBD

[∵ Angles in the same segments are equal]

Question 12:

Prove that a cyclic parallelogram is a rectangle.

Answer 12:



Given: Quadrilateral ABCD is a cyclic quadrilateral.

To prove: ABCD is a rectangle.

Proof: In cyclic quadrilateral ABCD

∠A + ∠C = 180° ... (1)

[∵ The sum of either pair of opposite angles of a cyclic quadrilateral is 180°.]

But ∠A = ∠C ... (2)

[∵ Opposite angles of a parallelogram are equal]

From the equation (1) and (2),

∠A + ∠A = 180°

⇒ 2∠A = 180°

$\Rightarrow \angle A = \frac{180^\circ}{2} = 90^\circ$

We know that, a parallelogram with one angle right angle, is a rectangle.

Hence, ABCD is a rectangle.