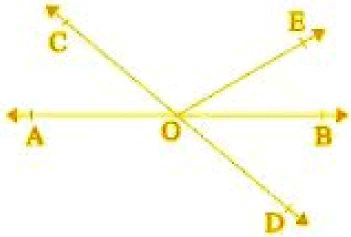
(Chapter – 6)(Lines and Angles)

(Class – 9)

Exercise 6.1

Question 1:

In Figure, lines AB and CD intersect at O. If ∠AOC + ∠BOE = 70° and ∠BOD = 40°, find ∠BOE and reflex ∠COE.



Answer 1:

Given: Lines AB and CD intersects at O such that ∠BOD = 40° and

∠AOC + ∠BOE = 70° ... (1)

∠AOC = ∠BOD [∵ Vertically Opposite Angles]

Hence, ∠AOC = 40° [∵ ∠BOD = 40°]

Therefore, from the equation (1), we have,

40°+ ∠BOE = 70°

⇒ ∠BOE = 70° − 40° = 30°

Here, ∠AOC + ∠BOE + ∠COE = 180° [∵ AOB is a straight line]

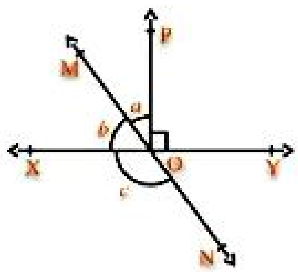
⇒ 70° + ∠COE = 180° [From the equation (1)]

⇒ ∠COE = 180° − 70° = 110° and

Reflex ∠COE = 360° − ∠COE = 360° − 110° = 250°

Question 2:

In Figure, lines XY and MN intersect at O. If ∠POY = 90° and $a : b = 2 : 3$, find c.



Answer 2:

Given: Lines XY and MN intersects at O, ∠POY = 90° and $a : b = 2 : 3$

Let, $a = 2x$, therefore $b = 3x$

Here, ∠XOM + ∠POM + ∠POY = 180° [∵ XOY is a straight line]

⇒ $3x + 2x + 90° = 180°$ [∵∠POY = 90°]

⇒ $5x + 90° = 180°$

⇒ $5x = 180° - 90° = 90°$

⇒ $x = \frac{180°}{5} = 18°$

Hence, ∠XOM = $3x = 3 \times 18° = 54°$ and

∠POM = $2x = 2 \times 18° = 36°$

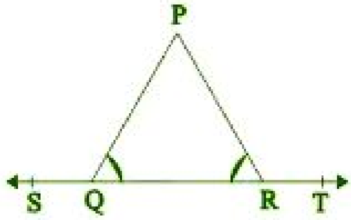
Here, ∠XON = ∠MOY [∵ Vertically Opposite Angles]

⇒ c = ∠POM + ∠POY [∵∠XON = c and ∠MOY = ∠POM + ∠POY]

⇒ $c = 36° + 90° = 126°$

Question 3:

In Figure, ∠PQR = ∠PRQ, then prove that ∠PQS = ∠PRT.



Answer 3:

∠PQS + ∠PQR = 180° [∵ Linear Pair] ... (1)

∠PRQ + ∠PRT = 180° [∵Linear Pair] ... (2)

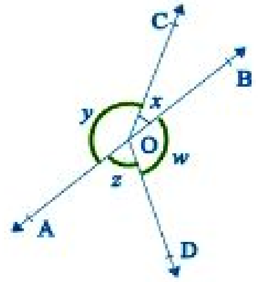
But, ∠PQR = ∠PRQ [∵ Given]

∴ From the equations (1) and (2), we have

∠PQS = ∠PRT

Question 4:

In Figure, if $x + y = w + z$, then prove that AOB is a line.



Answer 4:

We know that the sum of angles around a point is 360°. Therefore

$x + y + w + z = 360°$

$\Rightarrow (x + y) + (w + z) = 360°$

$\Rightarrow (x + y) + (x + y) = 360°$ $[\because$ Given $x + y = w + z]$

$\Rightarrow 2(x + y) = 360°$

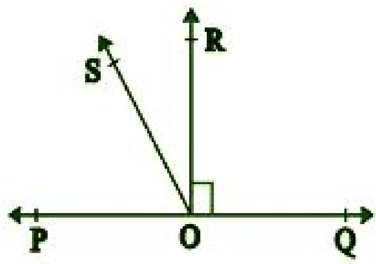
$\Rightarrow (x + y) = \frac{360°}{2}$

$\Rightarrow x + y = 180°$

∠AOC and ∠COB are forming linear pair. Hence, AOB is a straight line.

Question 5:

In Figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that: $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$



Answer 5:

$\text{RHS} = \frac{1}{2}(\angle QOS - \angle POS)$

$= \frac{1}{2}[(\angle QOR + \angle ROS) - (\angle POR - \angle ROS)]$

$[\because \angle QOS = \angle QOR + \angle ROS$ and $\angle POS = \angle POR - \angle ROS]$

$= \frac{1}{2}[\angle QOR + \angle ROS - \angle POR + \angle ROS]$

$= \frac{1}{2}[90° + \angle ROS - 90° + \angle ROS]$ $[\because \angle QOR = 90°$ और $\angle POR = 90°]$

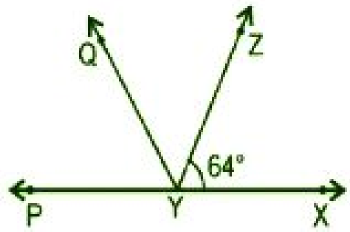
$= \frac{1}{2}[2\angle ROS]$

$= \angle ROS = \text{LHS}$

Question 6:

It is given that ∠XYZ = 64° and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects ∠ZYP, find ∠XYQ and reflex ∠QYP.

Answer 6:



$\angle PYZ + \angle XYZ = 180°$ $[\because$ Linear Pair$]$

$\Rightarrow \angle PYZ + 64° = 180°$ $[\because \angle XYZ = 64°]$

$\Rightarrow \angle PYZ = 180° - 64° = 116°$

But,

$\angle PYQ = \angle ZYQ = \frac{1}{2}\angle PYZ$ $[\because \angle ZYP$ is bisected by ray YQ$]$

$\therefore \angle PYQ = \angle ZYQ = \frac{1}{2} \times 116° = 58°$

$\therefore \angle XYQ = \angle XYZ + \angle ZYQ = 64° + 58° = 122°$ and

Reflex $\angle QYP = 360° - \angle QYP = 360° - 58° = 302°$

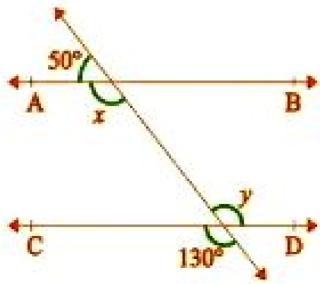
(Chapter – 6)(Lines and Angles)

(Class – 9)

Exercise 6.2

Question 1:

In Figure, find the values of x and y and then show that AB || CD.



Answer 1:

$50° + x = 180°$ [∵ Linear Pair]

$\Rightarrow x = 180° - 50° = 130°$ and

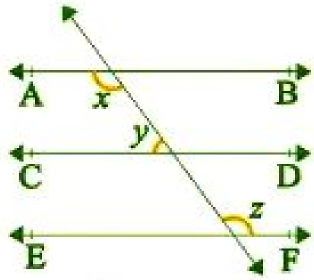
$y = 130°$ [∵ Vertically Opposite Angles]

Hence, $x = y = 130°$

Since, alternate angles are equal. Hence, AB || CD.

Question 2:

In Figure, if AB || CD, CD || EF and $y : z = 3 : 7$, find x.



Answer 2:

Given: $y : z = 3 : 7$

Let, $y = 3k$, therefore $z = 7k$

$y = \angle 1 = 3k$ [∵ Vertically Opposite Angles]

Given: CD || EF,

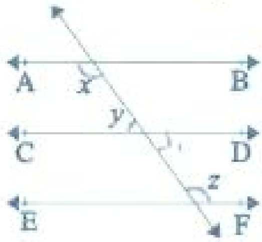
Therefore,

$\angle 1 + z = 180°$ [∵ Sum of co-interior angles]

$\Rightarrow 3k + 7k = 180°$

$\Rightarrow 10k = 180°$

$\Rightarrow k = \frac{180°}{10} = 18°$



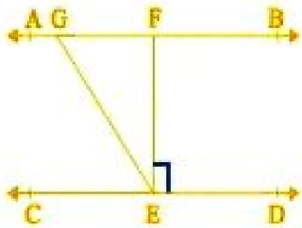
Hence, $z = 7k = 7 \times 18° = 126°$

Given that: AB || CD and CD || EF, therefore AB || EF

$\Rightarrow x = z = 126°$ [∵ Alternate Angles]

Question 3:

In Figure, if AB || CD, EF ⊥ CD and ∠GED = 126°, find ∠AGE, ∠GEF and ∠FGE.



Answer 3:

Given that: AB || CD,

Therefore,

$\angle AGE = \angle GED$ [∵ Alternate Angles]

$\Rightarrow \angle AGE = 126°$

From the figure,

$\angle GED = \angle GEF + \angle FED$

$\Rightarrow 126° = \angle GEF + 90°$

$\Rightarrow \angle GEF = 126° - 90° = 36°$

Given that: AB || CD,

Therefore,

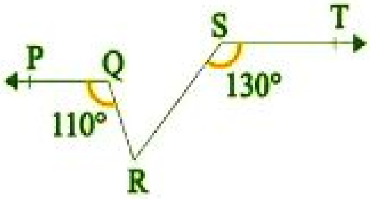
$\angle FGE + 126° = 180°$ [∵ Sum of co-interior angles]

$\Rightarrow \angle FGE = 180° - 126° = 54°$

Question 4:

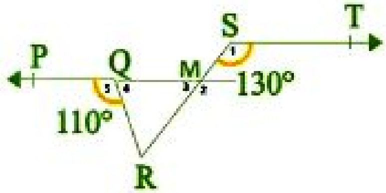
In Figure, if PQ || ST, ∠PQR = 110° and ∠RST = 130°, find ∠QRS.

[Hint: Draw a line parallel to ST through point R.]



Answer 4:

Construction: Produce PQ, so that it intersect ST at M.



Given that: PQ || ST, therefore

∠1 = ∠2 [∵ Corresponding Angles]

⇒ ∠2 = 130°

∠2+ ∠3 = 180° [∵ Linear Pair]

⇒ 130°+ ∠3 = 180°

⇒ ∠3 = 180° −130° = 50°

∠5+ ∠4 = 180° [∵ Linear Pair]

⇒ 110°+ ∠4 = 180°

⇒ ∠4 = 180° −110° = 70°

In triangle QMR,

∠3+ ∠4 +∠R = 180°

⇒ 50°+ 70° +∠R = 180°

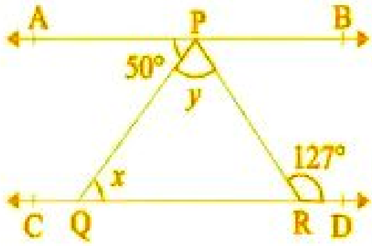
⇒ 120°+ ∠R = 180°

⇒ ∠R = 180° − 120° = 60°

Question 5:

In Figure, if AB || CD, ∠APQ = 50° and ∠PRD = 127°, find x and y.

Answer 5:



Given that: PQ || ST,

Therefore,

∠PQR = ∠APQ [∵ Alternate Angles]

⇒ x = 50°

∠APR = ∠PRD [∵ Alternate Angles]

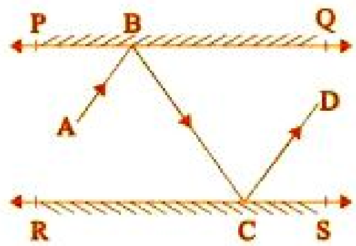
⇒ ∠APQ + ∠QPR = 127°

⇒ 50° + y = 127°

⇒ y = 127° − 50° = 77°

Question 6:

In Figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB || CD.



Answer 6:

Draw BE ⊥ PQ and CF ⊥ RS.

∠1 = ∠2 ... (i) [∵ Angle of incident = Angle of reflection]

Similarly,

∠3 = ∠4 ... (ii)

and,

∠2 = ∠3 ... (iii) [∵ Alternate Angles]

⇒ ∠1 = ∠4 [From the equations (i), (ii) and (iii)]

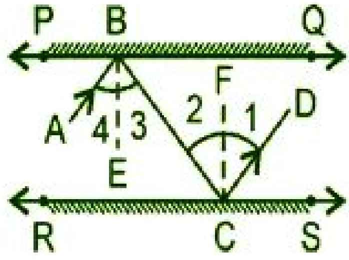
⇒ 2∠1 = 2∠4

⇒ ∠1 + ∠1 = ∠4 + ∠4

⇒ ∠1 + ∠2 = ∠3 + ∠4 [From the equation (i) and (ii)]

⇒ ∠BCD = ∠ABC

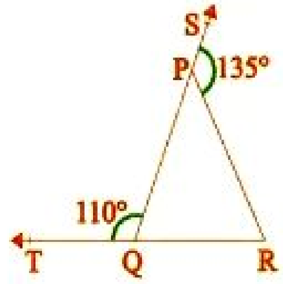
Since, the alternate angles are equal. Hence, AB || CD.



Exercise 6.3

Question 1:

In Figure, sides QP and RQ of ΔPQR are produced to points S and T respectively. If ∠SPR = 135° and ∠PQT = 110°, find ∠PRQ.

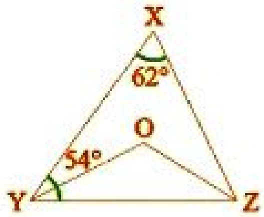


Answer 1:

∠PQT + ∠PQR = 180° [∵ Linear Pair]
⇒ 110°+ ∠PQR = 180°
⇒ ∠PQR = 180° − 110° = 70°
∠SPR + ∠QPR = 180° [∵ Linear Pair]
⇒ 135°+ ∠QPR = 180°
⇒ ∠QPR = 180° − 135° = 45°
In ΔPQR,
∠QPR + ∠PQR + ∠R = 180°
⇒ 70° + 45° + ∠R = 180°
⇒ 115° + ∠R = 180°
⇒ ∠R = 180° − 115° = 65°

Question 2:

In Figure, ∠X = 62°, ∠XYZ = 54°. If YO and ZO are the bisectors of ∠XYZ and ∠XZY respectively of ΔXYZ, find ∠OZY and ∠YOZ.



Answer 2:

Given that: ∠X = 62° and ∠XYZ = 54°
In ΔXYZ, ∠X + ∠XYZ + ∠XZY = 180°
⇒ 62° + 54° + ∠XZY = 180°
⇒ 116° + ∠XZY = 180°
⇒ ∠XZY = 180° − 116° = 64°
YO and ZO are the bisectors of ∠XYZ and ∠XZY respectively. therefore

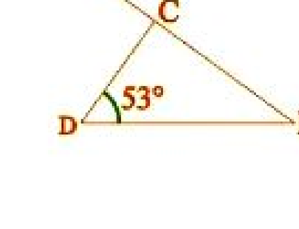
$$\angle OYZ = \frac{1}{2}\angle XYZ = \frac{1}{2} \times 54° = 27°$$

$$\angle OZY = \frac{1}{2}\angle XZY = \frac{1}{2} \times 64° = 32°$$

In ΔOYZ, ∠OZY + ∠OYZ + ∠YOZ = 180°
⇒ 32° + 27° + ∠YOZ = 180°
⇒ 59° + ∠YOZ = 180°
⇒ ∠YOZ = 180° − 59° = 121°

Question 3:

In Figure, if AB || DE, ∠BAC = 35° and ∠CDE = 53°, find ∠DCE.



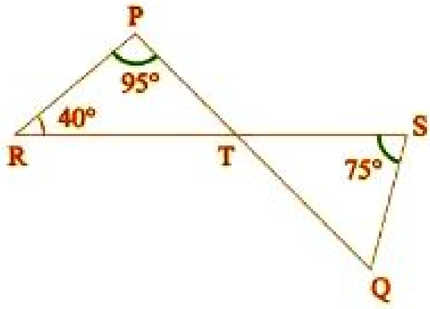
Answer 3:

Given that: AB || DE, therefore
∠CED = ∠BAC [∵ Alternate Angles]
⇒ ∠CED = 35°
In ΔCDE, ∠CED + ∠CDE + ∠DCE = 180°
⇒ 35° + 53° + ∠DCE = 180°
⇒ 88° + ∠DCE = 180°
⇒ ∠DCE = 180° − 88° = 92°

Question 4:

In Figure, if lines PQ and RS intersect at point T, such that ∠PRT = 40°, ∠RPT = 95° and ∠TSQ = 75°, find ∠SQT.

Answer 4:

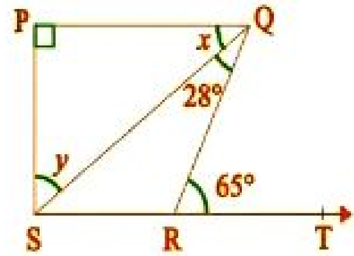


Given that: ∠PRT = 40°, ∠RPT = 95° and ∠TSQ = 75°
In ΔPTR, ∠P + ∠R + ∠PTR = 180°
⇒ 95° + 40° + ∠PTR = 180°
⇒ 135° + ∠PTR = 180°
⇒ ∠PTR = 180° − 135° = 45°
∠STQ = ∠PTR [∵ Vertically Opposite Angles]
⇒ ∠STQ = 45°
In ΔSQT, ∠STQ+ ∠S +∠SQT = 180°
⇒ 45°+ 75° +∠SQT = 180°
⇒ 120°+ ∠SQT = 180°
⇒ ∠SQT = 180° − 120° = 60°

Question 5:

In Figure, if PQ ⊥ PS, PQ || SR, ∠ SQR = 28° and ∠QRT = 65°, then find the values of x and y.

Answer 5:

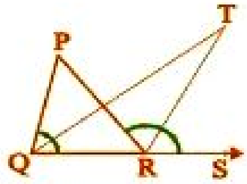


Given that: PQ ⊥ PS, PQ || SR, ∠SQR = 28° and ∠QRT = 65°
∠PQR = ∠QRT [∵ Alternate Angles]
⇒ ∠RQS + ∠PQS = 65°
⇒ 28° + x = 65°
⇒ x = 65° − 28° = 37°
In ΔPQS, ∠P + ∠PQS + ∠PSQ = 180°
⇒ 90°+ 37° + y = 180°
⇒ 127° + y = 180°
⇒ y = 180° − 127° = 53°

Question 6:

In Figure, the side QR of ΔPQR is produced to a point S. If the bisectors of ∠PQR and ∠PRS meet at point T, then prove that $\angle QTR = \frac{1}{2}\angle QPR$.

Answer 6:



∠PRS is the exterior angle of ΔPQR.
Therefore,
∠PRS = ∠QPR + ∠PQR

$$\Rightarrow \frac{1}{2}\angle PRS = \frac{1}{2}\angle QPR + \frac{1}{2}\angle PQR$$

$$\Rightarrow \angle TRS = \frac{1}{2}\angle QPR + \angle TQR \quad \dots (1) \quad [\because \angle TRS = \frac{1}{2}\angle PRS \text{ and } \angle TQR = \frac{1}{2}\angle PQR]$$

∠TRS is exterior angle of ΔTQR.
Therefore,
∠TRS = ∠QTR + ∠TQR ... (2)
From the equations (1) and (2), we have

$$\angle QTR + \angle TQR = \frac{1}{2}\angle QPR + \angle TQR$$

$$\Rightarrow \angle QTR = \frac{1}{2}\angle QPR$$