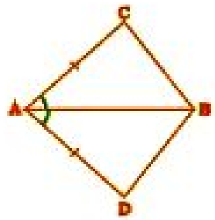
# Exercise 7.1

## Question 1:

In quadrilateral ACBD, AC = AD and AB bisects ∠A (see Figure). Show that ΔABC ≅ ΔABD. What can you say about BC and BD?




### Answer 1:

In ΔACB and ΔADB,

| | |
|---|---|
| AC = AD | [∵ Given] |
| ∠CAB = ∠DAB | [∵ AB bisects angle A] |
| AB = AB | [∵ Common] |
| अतः, ΔABC ≅ ΔABD | [∵ SAS Congruency Rule] |
| BC = BD | [∵ Corresponding parts of congruent triangles are equal] |

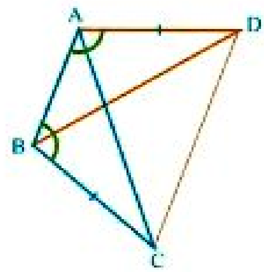
## Question 2:

ABCD is a quadrilateral in which AD = BC and ∠ DAB = ∠ CBA (see Figure). Prove that

**(i)** ΔABD ≅ ΔBAC

**(ii)** BD = AC

**(iii)** ∠ABD = ∠BAC



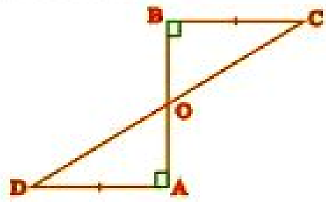

### Answer 2:

**(i)** In ΔABD and ΔBAC,

| | |
|---|---|
| AD = BC | [∵ Given] |
| ∠DAB = ∠CAB | [∵ Given] |
| AB = AB | [∵ Common] |
| Hence, ΔABD ≅ ΔBAC | [∵ SAS Congruency Rule] |
| **(ii)** BD = AC | [∵ Corresponding parts of congruent triangles are equal] |
| **(iii)** ∠ABD = ∠BAC | [∵ Corresponding parts of congruent triangles are equal] |

## Question 3:

AD and BC are equal perpendiculars to a line segment AB (see Figure). Show that CD bisects AB.




### Answer 3:

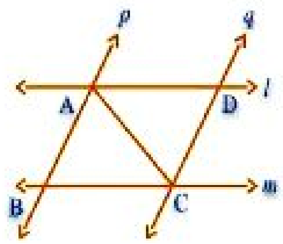
In ΔOCB and ΔODA,

| | |
|---|---|
| ∠BOC = ∠AOD | [∵ Vertically Opposite Angles] |
| ∠CBO = ∠DAO | [∵ Each 90°] |
| BC = AD | [∵ Given] |
| Hence, ΔOCB ≅ ΔODA | [∵ AAS Congruency Rule] |
| BO = AO | [∵Corresponding parts of congruent triangles are equal] |

Hence, CD bisects AB.

## Question 4:

*l* and *m* are two parallel lines intersected by another pair of parallel lines *p* and *q* (see Figure). Show that Δ ABC ≅ Δ CDA.




### Answer 4:

In ΔABC and ΔCDA,

| | |
|---|---|
| ∠BAC = ∠ACD | [∵ Alternate Angles] |
| AC = AC | [∵ Common] |
| ∠BCA = ∠DAC | [∵ Alternate Angles] |
| Hence, ΔABC ≅ ΔCDA | [∵ ASA Congruency Rule] |

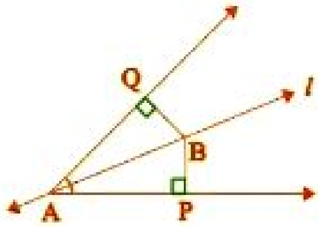
## Question 5:

Line $l$ is the bisector of an angle ∠A and B is any point on $l$. BP and BQ are perpendiculars from B to the arms of ∠A (see Figure). Show that:

**(i)** ΔAPB ≅ ΔAQB

**(ii)** BP = BQ or B is equidistant from the arms of ∠A.

### Answer 5:




**(i)** In ΔAPB and ΔAQB,

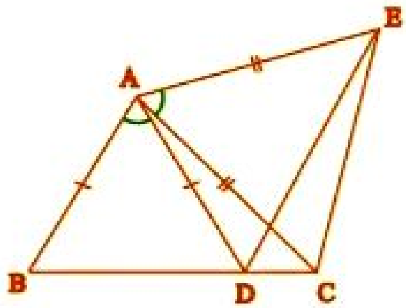
| | |
|---|---|
| ∠APB = ∠AQB | [∵ Each 90°] |
| ∠PAB = ∠QAB | [∵ Line $l$ bisects angle A] |
| AB = AB | [∵ Common] |
| Hence, ΔAPB ≅ ΔAQB | [∵ AAS Congruency Rule] |

**(ii)** BP = BQ [∵Corresponding parts of congruent triangles are equal]

## Question 6:

In Figure, AC = AE, AB = AD and ∠BAD = ∠EAC. Show that BC = DE.

### Answer 6:




∠BAD = ∠EAC [∵ Given]

Adding ∠DAC both sides, we have

∠BAD + ∠DAC = ∠EAC + ∠DAC

⇒ ∠BAC = ∠EAD

In ΔBAC and ΔDAE,

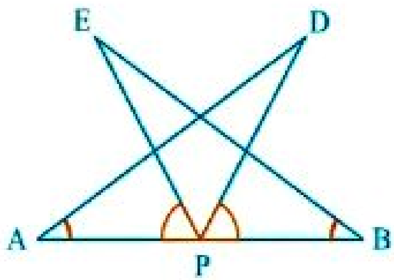
| | |
|---|---|
| AB = AD | [∵ Given] |
| ∠BAC = ∠EAD | [∵ Proved above] |
| AC = AE | [∵ Given] |
| Hence, ΔBAC ≅ ΔDAE | [∵ SAS Congruency Rule] |
| BC = DE | [∵Corresponding parts of congruent triangles are equal] |

## Question 7:

AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that ∠BAD = ∠ABE and ∠EPA = ∠DPB (see Figure). Show that

**(i)** ΔDAP ≅ ΔEBP

**(ii)** AD = BE




### Answer 7:

**(i)** ∠EPA =∠DPB [∵ Given]

Adding ∠EPD both sides, we have

∠EPA + ∠EPD = ∠DPB + ∠EPD

⇒ ∠APD = ∠BPE

In ΔDAP and ΔEBP,

| | |
|---|---|
| ∠A = ∠B | [∵ Given] |
| AP = PB | [∵ P is mid-point of line segment AB] |
| ∠APD = ∠BPE | [∵ Proved above] |
| Hence, ΔDAP ≅ ΔEBP | [∵ ASA Congruency Rule] |

**(ii)** AD = BE [∵Corresponding parts of congruent triangles are equal]

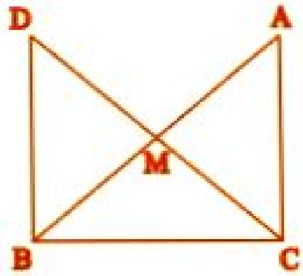
## Question 8:

In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see Figure). Show that:

**(i)** ΔAMC ≅ ΔBMD

**(ii)** ∠DBC is a right angle.

**(iii)** ΔDBC ≅ ΔACB




## Answer 8:

**(i)** In ΔAMC and ΔBMD,

| | |
|---|---|
| CM = DN | [∵ Given] |
| ∠AMC = ∠BMD | [∵ Vertically Opposite Angles] |
| AM = MB | [∵ M is the mid-point of line segment AB] |
| Hence, ΔAMC ≅ ΔBMD | [∵ SAS Congruency Rule] |

| | |
|---|---|
| **(ii)** ΔAMC ≅ ΔBMD | [∵ Proved above] |
| ∠CAM = ∠DBM | [∵Corresponding parts of congruent triangles are equal] |

Since, alternate angles (∠CAM and ∠DBM) are equal, therefore AC || BD.

| | |
|---|---|
| ∠ACB + ∠DBC = 180° | [∵ Co-interior angles] |
| ⇒ 90° + ∠DBC = 180° | [∵ Angle C is right angle] |
| ⇒ ∠DBC = 180° − 90° = 90° | |

Hence, ∠DBC is a right angle.

**(iii)** In ΔDBC and ΔACB,

| | |
|---|---|
| DB = AC | [∵ ΔAMC ≅ ΔBMD] |
| ∠DBC = ∠ACB | [∵ Proved above] |
| BC = BC | [∵ M is the mid-point of line segment AB] |
| Hence, ΔDBC ≅ ΔACB | [∵ SAS Congruency Rule] |

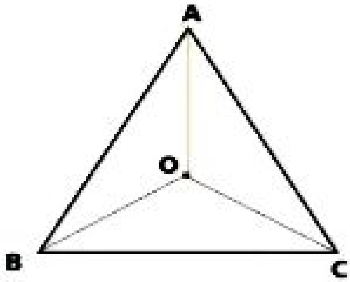
# Exercise 7.2

## Question 1:

In an isosceles triangle ABC, with AB = AC, the bisectors of ∠B and ∠C intersect each other at O. Join A to O. Show that:

**(i)** OB = OC

**(ii)** AO bisects ∠A



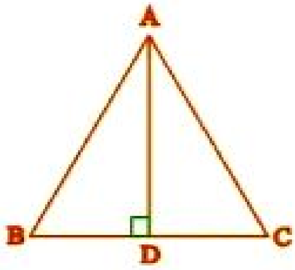

### Answer 1:

| | |
|---|---|
| **(i)** In ABC, AB = AC | [∵ Given] |
| Hence, ∠ACB = ∠ABC | [∵ Angles opposite to equal sides are equal] |
| ⇒ $\frac{1}{2}\angle ACB = \frac{1}{2}\angle ABC$ | |
| ⇒ ∠ACO = ∠ABO | [∵ OB and OC bisect ∠B and ∠C respectively] |
| In ΔABO and ΔACO, | |
| AB = AC | [∵ Given] |
| ∠ABO = ∠ACO | [∵ Proved above] |
| AO = AO | [∵ Common] |
| Hence, ΔABO ≅ ΔACO | [∵ SAS Congruency Rule] |
| OB = OC | [∵ CPCT] |
| **(ii)** ΔABO ≅ ΔACO | [∵ Proved above] |
| ∠BAO = ∠CAO | [∵ CPCT] |

Hence, OA bisects angle A.

## Question 2:

In ΔABC, AD is the perpendicular bisector of BC (see Figure). Show that ΔABC is an isosceles triangle in which AB = AC.



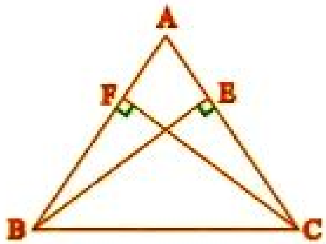

### Answer 2:

| | |
|---|---|
| In ΔABD and ΔACD, | |
| BD = DC | [∵ AD bisects BC] |
| ∠ADB = ∠ADC | [∵ Each 90°] |
| AD = AD | [∵ Common] |
| Hence, ΔABD ≅ ΔACD | [∵ SAS Congruency Rule] |
| AB = AC | [∵ CPCT] |

Hence, ΔABC is an isosceles triangle.

## Question 3:

ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see Figure). Show that these altitudes are equal.




### Answer 3:

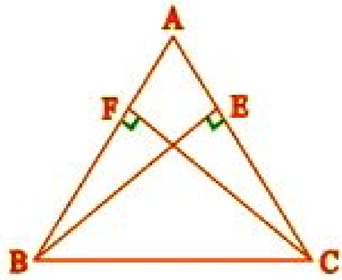
| | |
|---|---|
| In ΔABE and ΔACF, | |
| ∠AEB = ∠AFC | [∵ Each 90°] |
| ∠A = ∠A | [∵ Common] |
| AB = AC | [∵ Given] |
| Hence, ΔABE ≅ ΔACF | [∵ AAS Congruency Rule] |
| BE = CF | [∵ CPCT] |

## Question 4:

ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see Figure). Show that

**(i)** $\Delta ABE \cong \Delta ACF$

**(ii)** AB = AC, i.e., ABC is an isosceles triangle.




## Answer 4:

**(i)** In $\Delta ABE$ and $\Delta ACF$,

$\angle AEB = \angle AFC$ [∵ Each 90°]

$\angle A = \angle A$ [∵ Common]

BE = CF [∵ Given]

Hence, $\Delta ABE \cong \Delta ACF$ [∵ AAS Congruency Rule]

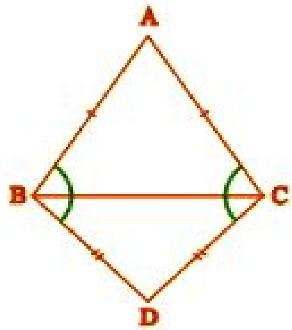
**(ii)** In $\Delta ABE \cong \Delta ACF$ [∵ Proved above]

AB = AC [∵ CPCT]

Hence, $\Delta ABC$ is an isosceles triangle.

## Question 5:

ABC and DBC are two isosceles triangles on the same base BC (see Figure). Show that $\angle ABD = \angle ACD$.




## Answer 5:

In $\Delta ABC$,

AB = AC [∵ Given]

$\angle ABC = \angle ACB$ ... (1) [∵ Angles opposite to equal sides are equal]

In $\Delta DBC$,

DB = DC [∵ Given]

$\angle DBC = \angle DCB$ ... (2) [∵ Angles opposite to equal sides are equal]

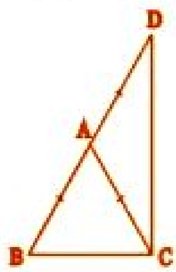
Adding equation (1) and (2), we get

$\angle ABC + \angle DBC = \angle ACB + \angle DCB$

$\Rightarrow \angle ABD = \angle ACD$

## Question 6:

$\Delta ABC$ is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB (see Figure). Show that $\angle BCD$ is a right angle.




## Answer 6:

In $\Delta ACD$,

AB = AC [∵ Given]

$\angle ACD = \angle D$ ... (1) [∵ Angles opposite to equal sides are equal]

In $\Delta ABC$,

AB = AC [∵ Given]

$\angle B = \angle ACB$ ... (2) [∵ Angles opposite to equal sides are equal]

In $\Delta DBC$,

$\angle D + \angle B + \angle BCD = 180°$

$\Rightarrow \angle ACD + \angle ACB + \angle BCD = 180°$ [∵ From the equation (1) and (2)]

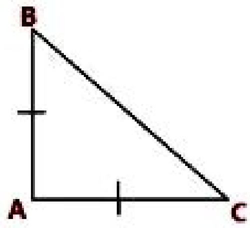
$\Rightarrow \angle BCD + \angle BCD = 180°$ [∵ $\angle ACD + \angle ACB = \angle BCD$]

$\Rightarrow 2\angle BCD = 180°$

$\Rightarrow \angle BCD = \frac{180°}{2} = 90°$

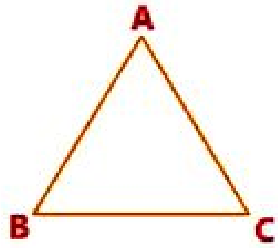
Hence, $\angle BCD$ is a right angle.

## Question 7:
ABC is a right angled triangle in which ∠A = 90° and AB = AC. Find ∠B and ∠C.




### Answer 7:
In ΔABC,
AB = AC [∵ Given]
∠B = ∠C [∵ Angles opposite to equal sides are equal]
In ΔABC,
∠A + ∠B + ∠C = 180°
⇒ 90° + ∠B + ∠C = 180° [∵ ∠A = 90°]
⇒ 90° +∠B + ∠B = 180° [∵∠C = ∠B]
⇒ 2∠B = 180° − 90° = 90°
⇒ ∠B = $\frac{90°}{2}$ = 45°
Hence, ∠B = ∠C = 90°

## Question 8:
Show that the angles of an equilateral triangle are 60° each.




### Answer 8:
In ΔABC,
AB = AC [∵ Given]
∠C = ∠B ... (1) [∵ Angles opposite to equal sides are equal]
Similarly,
In ΔABC,
AB = BC [∵ Given]
∠C = ∠A ... (2) [∵ Angles opposite to equal sides are equal]
From the equation (1) and (2), we have
∠A = ∠B = ∠C ... (3)
In ΔABC,
∠A + ∠B + ∠C = 180°
⇒∠A + ∠A + ∠A = 180° [∵ From the equaion (3)]
⇒ 3∠A = 180°
⇒ ∠A = $\frac{180°}{3}$ = 60°
Hence, ∠A = ∠B = ∠C = 60°

# (Chapter – 7)(Triangles)

**(Class – 9)**

## Exercise 7.3

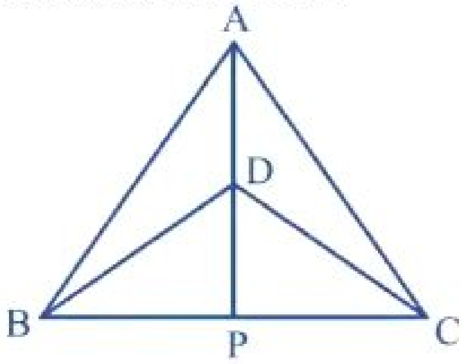
### Question 1:

$\Delta ABC$ and $\Delta DBC$ are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see Figure). If AD is extended to intersect BC at P, show that

**(i)** $\Delta ABD \cong \Delta ACD$ **(ii)** $\Delta ABP \cong \Delta ACP$

**(iii)** AP bisects $\angle A$ as well as $\angle D$. **(iv)** AP is the perpendicular bisector of BC.

### Answer 1:




**(i)** In $\Delta ABD$ and $\Delta ACD$,

$AB = AC$ [$\because$ Given]

$BD = CD$ [$\because$ Given]

$AD = AD$ [$\because$ Common]

Hence, $\Delta ABD \cong \Delta ACD$ [$\because$ SSS Congruency Rule]

**(ii)** In $\Delta ABD \cong \Delta ACD$ [$\because$ Proved above]

$\angle BAD = \angle CAD$ [$\because$ CPCT]

In $\Delta ABP$ and $\Delta ACP$,

$AB = AC$ [$\because$ Given]

$\angle BAP = \angle CAP$ [$\because$ Proved above]

$AP = AP$ [$\because$ Common]

Hence, $\Delta ABP \cong \Delta ACP$ [$\because$ SAS Congruency Rule]

**(iii)** In $\Delta ABD \cong \Delta ACD$ [$\because$ Proved above]

$\angle BAD = \angle CAD$ [$\because$ CPCT]

$\angle BDA = \angle CDA$ [$\because$ CPCT]

Hence, AP bisects both the angles A and D.

**(iv)** In $\Delta ABP \cong \Delta ACP$ [$\because$ Proved above]

$BP = CP$ [$\because$ CPCT]

$\angle BPA = \angle CPA$ [$\because$ CPCT]

$\angle BPA + \angle CPA = 180°$ [$\because$ Linear Pair]

$\Rightarrow \angle CPA + \angle CPA = 180°$ [$\because \angle BPA = \angle CPA$]

$\Rightarrow 2\angle CPA = 180°$ $\Rightarrow \angle CPA = \frac{180°}{2} = 90°$

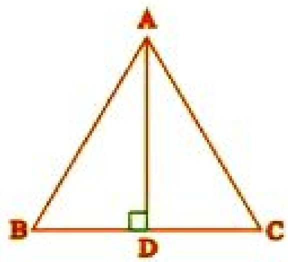
$\Rightarrow$ AP is perpendicular to BC. $\Rightarrow$ AP is perpendicular bisector of BC. [$\because BP = CP$]

### Question 2:

AD is an altitude of an isosceles triangle ABC in which $AB = AC$. Show that

**(i)** AD bisects BC **(ii)** AD bisects $\angle A$.

### Answer 2:




**(i)** In $\Delta ABD$ and $\Delta ACD$,

$\angle ADB = \angle ADC$ [$\because$ Each 90°]

$AB = AC$ [$\because$ Given]

$AD = AD$ [$\because$ Common]

Hence, $\Delta ABD \cong \Delta ACD$ [$\because$ RHS Congruency Rule]

$BD = DC$ [$\because$ CPCT]

Hence, AD bisects BC.

**(ii)** $\angle BAD = \angle CAD$ [$\because$ CPCT]

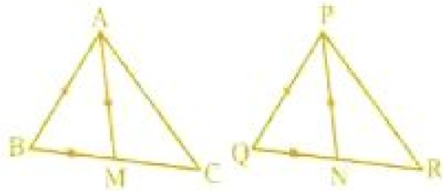
Hence, AD bisects angle A.

## Question 3:

Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR (see Figure). Show that:

**(i)** ΔABM ≅ ΔPQN

**(ii)** ΔABC ≅ ΔPQR




### Answer 3:

**(i)** Given that: BC = QR

$\Rightarrow \frac{1}{2}BC = \frac{1}{2}QR \Rightarrow BM = QN$ [∵ AM and PN are medians]

In ΔABM and ΔPQN,

AB = PQ [∵ Given]

AM = PN [∵ Given]

BM = QN [∵ Proved Above]

Hence, ΔABM ≅ ΔPQN [∵ SSS Congruency Rule]

**(ii)** In ΔABM ≅ ΔPQN [∵ Prvoed Above]

∠B = ∠Q [∵ CPCT]

In ΔABC and ΔPQR,

AB = PQ [∵ Given]

∠B = ∠Q [∵ Proved Above]

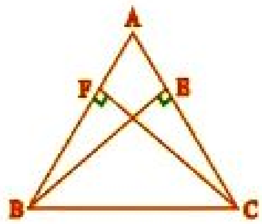
BC = QR [∵ Given]

Hence, ΔABC ≅ ΔPQR [∵ SAS Congruency Rule]

## Question 4:

BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

### Answer 4:




In ΔFBC and ΔECB,

∠BFC = ∠CEB [∵ Each 90°]

BC = BC [∵ Common]

FC = BE [∵ Given]

Hence, ΔFBC ≅ ΔECB [∵ RHS Congruency Rule]

∠FBC = ∠ECB [∵ CPCT]

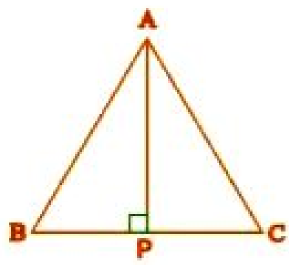
⇒ AC = AB [∵ Angles opposite to equal sides are equal]

Hence, ΔABC is an isosceles triangle.

## Question 5:

ABC is an isosceles triangle with AB = AC. Draw AP ⊥ BC to show that ∠B = ∠C.

### Answer 5:




In ΔABP and ΔACP,

∠APB = ∠APC [∵ Each 90°]

AB = AC [∵ Given]

AP = AP [∵ Common]

Hence, ΔABP ≅ ΔACP [∵ RHS Congruency Rule]

∠B = ∠C [∵ CPCT]

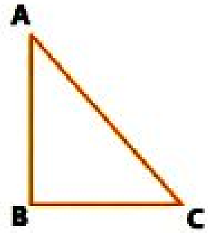
## (Chapter – 7)(Triangles)

**(Class – 9)**

## Exercise 7.4

### Question 1:

Show that in a right angled triangle, the hypotenuse is the longest side.




**Answer 1:**

In $\Delta ABC$, $\angle B = 90^\circ$ [∵ Given]

Therefore, $\angle A < 90^\circ$ and $\angle C < 90^\circ$ [$\because \angle A + \angle C = 90^\circ$]

Hence, in $\Delta ABC$,

$\angle B > \angle C$ [$\because \angle B = 90^\circ$ and $\angle C < 90^\circ$]

$AC > AB$ ... (1) [∵ In a triangle, greater angle has longer side opposite to it]

Similarly, in $\Delta ABC$, $\angle B > \angle A$ [$\because \angle B = 90^\circ$ and $\angle A < 90^\circ$]

$AC > BC$ ... (2) [∵ In a triangle, greater angle has longer side opposite to it]

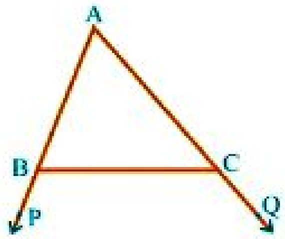
From the equation (1) and (2), we have

$AC > AB$ and $AC > BC$

Hence, hypotenuse AC is the longest side.

### Question 2:

In Figure, sides AB and AC of $\Delta ABC$ are extended to points P and Q respectively. Also, $\angle PBC < \angle QCB$. Show that $AC > AB$.




**Answer 2:**

$\angle PBC + \angle ABC = 180^\circ$ [∵ Linear Pair]

$\Rightarrow \angle PBC = 180^\circ - \angle ABC$ ... (1)

Similarly,

$\angle QCB + \angle ACB = 180^\circ$ [∵ Linear Pair]

$\Rightarrow \angle QCB = 180^\circ - \angle ACB$ ... (2)

Given that:

$\angle PBC < \angle QCB$

$\Rightarrow 180^\circ - \angle ABC < 180^\circ - \angle ACB$ [∵ From the equation (1) and (2)]

$\Rightarrow -\angle ABC < -\angle ACB$

$\Rightarrow \angle ABC > \angle ACB$ ... (3)

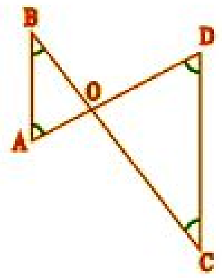
In $\Delta ABC$,

$\angle ABC > \angle ACB$ [∵ From the equation (3)]

Hence, $AC > AB$ [∵ In a triangle, greater angle has longer side opposite to it]

### Question 3:

In Figure, $\angle B < \angle A$ and $\angle C < \angle D$. Show that $AD < BC$.




**Answer 3:**

In $\Delta ABO$,

$\angle B < \angle A$ [∵ Given]

Hence, $AO < BO$ ... (1) [∵ In a triangle, greater angle has longer side opposite to it]

Similarly,

In $\Delta CDO$,

$\angle C < \angle D$ [∵ Given]

Hence, $DO < CO$ ... (2) [∵ In a triangle, greater angle has longer side opposite to it]

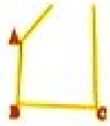
From the equation (1) and (2), we have

$AO + DO < BO + CO$

$\Rightarrow AD < BC$

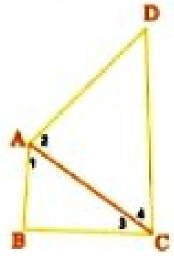
## Question 4:

AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (see Fig...). Show that ∠ A > ∠ C and ∠ B > ∠ D.



### Answer 4:

**Construction**: Join AC.




In ΔABC, BC > AB  [∵ AB is the shortest side of the quadrilateral ABCD]

Hence, ∠1 > ∠3  ... (1)  [∵ In a triangle, longer side has greater angle opposite to it]

Similarly,

In ΔADC,

CD > AD  [∵ CD is the longest side of the quadrilateral ABCD]

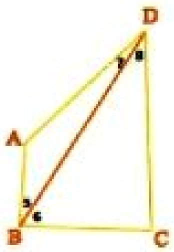
Hence, ∠2 > ∠4  ... (2)  [∵ In a triangle, longer side has greater angle opposite to it]

From the equation (1) and (2), we have

∠1 + ∠2 > ∠3 + ∠4

⇒ ∠A > ∠C

**Construction**: Join BD.




In ΔABD, AD > AB  [∵ AB is the shortest side of the quadrilateral ABCD]

Hence, ∠5 > ∠7  ... (3)  [∵ In a triangle, longer side has greater angle opposite to it]

Similarly,

In ΔBDC, CD > BC  [∵ CD is the longest side of the quadrilateral ABCD]

Hence, ∠6 > ∠8  ... (4)  [∵ In a triangle, longer side has greater angle opposite to it]

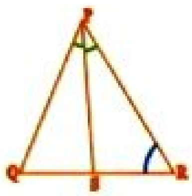
From the equation (3) and (4), we have

∠5 + ∠6 > ∠7 + ∠8 ⇒ ∠B > ∠D

## Question 5:

In Figure, PR > PQ and PS bisects ∠QPR. Prove that ∠PSR > ∠PSQ.

### Answer 5:




In ΔPQR,

PR > PQ  [∵ Given]

Hence, ∠Q > ∠R  [∵ In a triangle, longer side has greater angle opposite to it]

Adding ∠RPS both sides,

∠Q + ∠RPS > ∠R + ∠RPS

⇒∠Q + ∠RPS > ∠PSQ  [∵∠PSQ is exterior angle of triangle PSR]

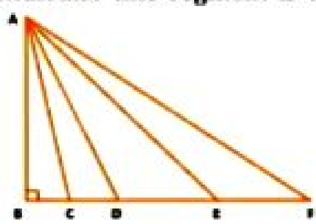
⇒∠Q + ∠QPS > ∠PSQ  [∵∠RPS =∠QPS]

⇒∠PSR > ∠PSQ  [∵∠PSR is exterior angle of triangle PQS]

## Question 6:

Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

### Answer 6:




Given: FB is a line and A is a point outside of FB.

To Prove: AB is smallest line segment.

In ΔABC, ∠B = 90°  [∵ Given]

Therefore, ∠BAC < 90° and ∠ACB < 90°  [∵∠BAC + ∠ACB = 90°]

Hence, in ΔABC, ∠B > ∠ACB  [∵∠B = 90° and ∠ACB < 90°]

AC > AB  [∵In a triangle, greater angle has longer side opposite to it]

Similarly, AD > AB, AE > AB and AF > AB,

Hence, AB is the smallest line.