Exercise 8.1

Question 1:

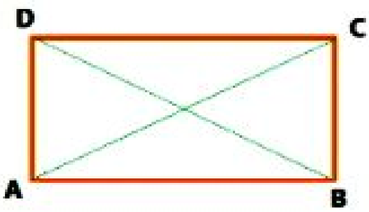
The angles of quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

Answer 1:

Let the first angle = $3x$
Therefore, the second angle = $5x$,
Third angle = $9x$ and
Fourth angle = $13x$
Sum of all angles of a quadrilateral is 360°. Therefore, $3x + 5x + 9x + 13x = 360°$
$\Rightarrow 30x = 360° \quad \Rightarrow x = \frac{360°}{30} = 12°$
Hence, the first angle = $3 \times 12° = 36°$,
The second angle = $5 \times 12° = 60°$,
Third angle = $9 \times 12° = 108°$
The forth angle = $13 \times 12° = 156°$

Question 2:

If the diagonals of a parallelogram are equal, then show that it is a rectangle.



Answer 2:

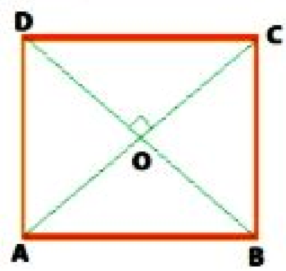
Given: ABCD is a parallelogram with AC = BD.
To Prove: ABCD is a rectangle.
Solution: In ΔABC and ΔBAD,

BC = AD	[∵ Opposite sides of a parallelogram are equal]
AC = BD	[∵ Given]
AB = AB	[∵ Common]
Hence, ΔABC ≅ ΔBAD	[∵ SSS Congruency rule]
∠ABC = ∠BAD	[∵ CPCT]
But, ∠ABC + ∠BAD = 180°	[∵ Co-interior angles]
⇒ 2∠BAD = 180°	[∵ ∠ABC = ∠BAD]

$\Rightarrow \angle BAD = \frac{180°}{2} = 90°$
A parallelogram with one of its angle is 90° is a rectangle. Hence, ABCD is a rectangle.

Question 3:

Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.



Answer 3:

Given: ABCD is a quadrilateral in which AO = CO, BO = DO and ∠COD = 90°.
To prove: ABCD is a rhombus.
Solution: In ΔAOB and ΔAOD,

BO = DO	[∵ Given]
∠AOB = ∠AOD	[∵ Each 90°]
AO = AO	[∵ Common]
Hence, ΔAOB ≅ ΔAOD	[∵ SAS Congruency rule]
AB = AD	[∵ CPCT]

Similarly, AB = BC and BC = CD
Now, all the four sides of quadrilateral ABCD are equal.
Hence, ABCD is a rhombus.

Question 4:

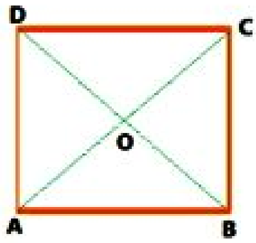
Show that the diagonals of a square are equal and bisect each other at right angles.

Answer 4:

Given: ABCD is a square.

To prove: $AC = BD$, $AO = CO$, $BO = DO$ and $\angle COD = 90^\circ$.

Solution: ΔBAD and ΔABC,



$AD = BC$	[$\because$ Opposite sides of a square]
$\angle BAD = \angle ABC$	[$\because$ Each 90°]
$AB = AB$	[$\because$ Common]
Hence, $\Delta BAD \cong \Delta ABC$	[$\because$ SAS Congruency rule]
$BD = AC$	[$\because$ CPCT]
In ΔAOB and ΔCOD,	
$\angle OAB = \angle OCD$	[$\because$ Alternate angles]
$AB = CD$	[$\because$ Opposite sides of a square]
$\angle OBA = \angle ODC$	[$\because$ Alternate angles]
Hence, $\Delta BAD \cong \Delta ABC$	[$\because$ ASA Congruency rule]
$AO = OC$, $BO = OD$	[$\because$ CPCT]
In ΔAOB and ΔAOD,	
$OB = OD$	[$\because$ Proved above]
$AB = AD$	[$\because$ Sides of a square]
$OA = OA$	[$\because$ Common]
Hence, $\Delta BAD \cong \Delta ABC$	[$\because$ SSS Congruency rule]
$\angle AOB = \angle AOD$	[$\because$ CPCT]
But, $\angle AOB + \angle AOD = 180^\circ$	[$\because$ Linear Pair]
$\Rightarrow 2\angle AOB = 180^\circ$	[$\because \angle AOD = \angle AOB$]
$\Rightarrow \angle AOB = \frac{180^\circ}{2} = 90^\circ$	

Hence, the diagonals of a square are equal and bisect each other at right angles.

Question 5:

Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Answer 5:

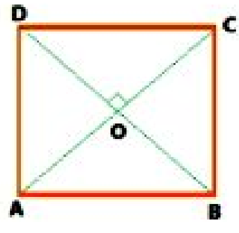
Given: ABCD is a quadrilateral such that $AC = BD$, $AO = CO$, $BO = DO$ and $\angle COD = 90^\circ$.

To prove: ABCD is a square.

Solution: If the diagonals of a quadrilateral bisects each other at right angle, it is a rhombus.

Hence, $AB = BC = CD = DA$

In ΔBAD and ΔABC,

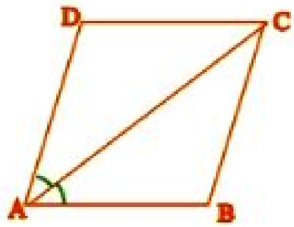


$AD = BC$	[$\because$ Proved above]
$BD = AC$	[$\because$ Given]
$AB = AB$	[$\because$ Common]
Hence, $\Delta BAD \cong \Delta ABC$	[$\because$ SSS Congruency rule]
$\angle BAD = \angle ABC$	[$\because$ CPCT]
But, $\angle BAD + \angle ABC = 180^\circ$	[$\because$ Co-interior angles]
$\Rightarrow 2\angle ABC = 180^\circ$	[$\because \angle BAD = \angle ABC$]
$\Rightarrow \angle ABC = \frac{180^\circ}{2} = 90^\circ$	

Hence, if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Question 6:

Diagonal AC of a parallelogram ABCD bisects ∠A (see Figure). Show that
(i) it bisects ∠C also, **(ii)** ABCD is a rhombus.



Answer 6:

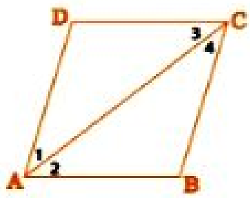
(i) ∠DAC = ∠BAC ... (1) [∵ Given]
∠DAC = ∠BCA ... (2) [∵ Alternate angles]
∠BAC = ∠ACD ... (3) [∵ Alternate angles]
From the equations (1), (2) and (3), we have
∠ACD = ∠BCA ... (4)
Hence, diagonal AC bisects angle C also.

(ii) From the equation (2) and (4), we have
∠ACD = ∠DAC
In ΔADC,
∠ACD = ∠DAC [∵ Proved above]
AD = DC [∵ In a triangle, the sides opposite to equal angle are equal]
A parallelogram whose adjacent sides are equal, is a rhombus. Hence, ABCD is a rhombus.

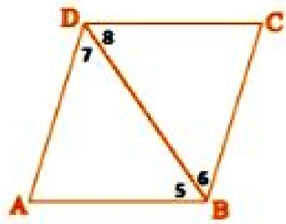
Question 7:

ABCD is a rhombus. Show that diagonal AC bisects ∠A as well as ∠C and diagonal BD bisects ∠B as well as ∠D.

Answer 7:



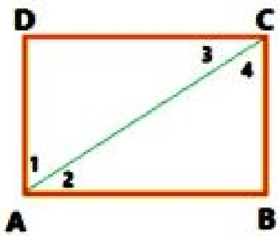
In ΔADC,
AD = DC [∵ ABCD is a rhombus]
∠3 = ∠1 ... (1) [∵ Angles opposite to equal sides are equal]
But, ∠3 = ∠2 ... (2) [∵ Alternate angles]
Hence, ∠1 = ∠2 ... (3) [∵ From (1) and (2)]
and ∠1 = ∠4 ... (4) [∵ Alternate angles]
Hence, ∠3 = ∠4 ... (5) [∵ From (1) and (4)]
Hence, from (3) and (5), diagonal AC bisects angle A as well as angle C.



In ΔADB,
AD = AB [∵ ABCD is a rhombus]
∠5 = ∠7 ... (6) [∵ Angles opposite to equal sides are equal]
But, ∠7 = ∠6 ... (7) [∵ Alternate angles]
Hence, ∠5 = ∠6 ... (8) [∵ From (6) and (7)]
and ∠5 = ∠8 ... (9) [∵ Alternate angles]
Hence, ∠7 = ∠8 ... (10) [∵ From (6) and (9)]
Hence, from (8) and (10), diagonal BD bisects angle B as well as angle D.

Question 8:

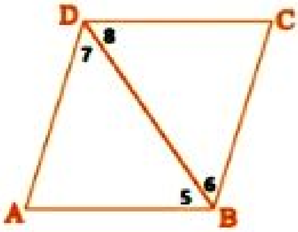
ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C. Show that:
(i) ABCD is a square **(ii)** diagonal BD bisects ∠B as well as ∠D.



Answer 8:

(i) Given: ABCD is a rectangle ∠1 = ∠2 and ∠3 = ∠4.
To prove: ABCD is a square.
Solution: ∠1 = ∠4 ... (1) [∵ Alternate angles]
∠3 = ∠4 ... (2) [∵ Given]
अतः, ∠1 = ∠3 ... (3) [∵ From (1) and (2)]
In ΔADC,

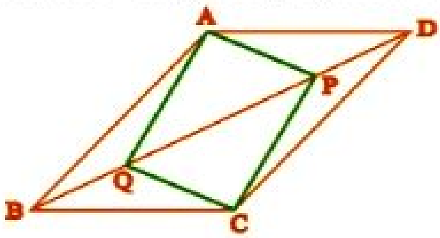
$\angle 1 = \angle 3$ [∵ From (3)]
DC = AD [∵ In a triangle, sides opposite to equal angle are equal]
A rectangle, whose adjacent sides are equal, is a square.
Hence, ABCD is a square.



(ii) To prove: Diagonal BD bisects angle B as well as angle D.
Solution: $\angle 5 = \angle 8$ ... (4) [∵ Alternate angles]
In ΔADB,
AB = AD [∵ ABCD is a square]
$\angle 7 = \angle 5$ ... (5) [∵ Angles opposite to equal sides are equal]
Hence, $\angle 7 = \angle 8$ ... (6) [∵ From (4) and (5)]
and $\angle 7 = \angle 6$ ... (7) [∵ Alternate angles]
Hence, $\angle 5 = \angle 6$ ... (8) [∵ From (5) and (7)]
Hence, from (6) and (8), diagonal BD bisects angle B as well as D.

Question 9:

In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see Figure). Show that:
(i) $\Delta APD \cong \Delta CQB$
(ii) AP = CQ
(iii) $\Delta AQB \cong \Delta CPD$
(iv) AQ = CP
(v) APCQ is a parallelogram



Answer 9:

(i) In ΔAPD and ΔCQB,
DP = BQ [∵ Given]
$\angle ADP = \angle CBQ$ [∵ Alternate angles]
AD = BC [∵ Opposite sides of a parallelogram]
Hence, $\Delta APD \cong \Delta CQB$ [∵ SAS Congruency rule]

(ii) $\Delta APD \cong \Delta CQB$ [∵ Proved above]
AP = CQ ... (1) [∵ CPCT]

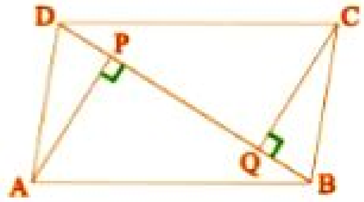
(iii) In ΔAQB and ΔCPD,
QB = DP [∵ Given]
$\angle ABQ = \angle CDP$ [∵ Alternate angles]
AB = CD [∵ Opposite sides of a parallelogram]
Hence, $\Delta AQB \cong \Delta CPD$ [∵ SAS Congruency rule]

(iv) $\Delta AQB \cong \Delta CPD$ [∵ Proved above]
AQ = CP ... (2) [∵ CPCT]

(v) In APCQ,
AP = CQ [∵ From (1)]
AQ = CP [∵ From (2)]
The opposite sides of quadrilateral APCQ are equal.
Hence, APCQ is a parallelogram.

Question 10:

ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see Figure).
Show that: (i) ΔAPB ≅ ΔCQD (ii) AP = CQ



Answer 10:

(i) In ΔAPB and ΔCQD,

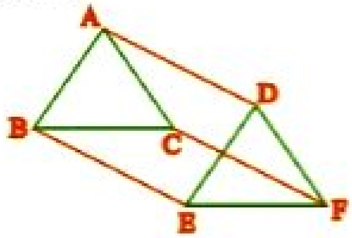
∠APB = ∠CQD [∵ Each 90°]
∠ABP = ∠CDQ [∵ Alternate angles]
AB = CD [∵ Opposite sides of a parallelogram]
Hence, ΔAPB ≅ ΔCQD [∵ SAS Congruency rule]

(ii) ΔAPB ≅ ΔCQD [∵ Proved above]
AP = CQ [∵ CPCT]

Question 11:

In ΔABC and ΔDEF, AB = DE, AB || DE, BC = EF and BC || EF. Vertices A, B and C are joined to vertices D, E and F respectively (see Figure). Show that

(i) quadrilateral ABED is a parallelogram
(ii) quadrilateral BEFC is a parallelogram
(iii) AD || CF and AD = CF
(iv) quadrilateral ACFD is a parallelogram
(v) AC = DF
(vi) ΔABC ≅ ΔDEF.



Answer 11:

(i) In ABED, AB = DE [∵ Given]
AB || DE [∵ Given]
Hence, ABED is a parallelogram.

(ii) In BEFC, BC = EF [∵ Given]
BC || EF [∵ Given]
Hence, BEFC is a parallelogram.

(iii) In ABED,
AD = BE ... (1) [∵ ABED is a parallelogram]
AD || BE ... (2) [∵ ABED is a parallelogram]
In BEFC,
BE = CF ... (3) [∵ ABED is a parallelogram]
BE || CF ... (4) [∵ ABED is a parallelogram]
From (2) and (4), we have
AD || CF ... (5)
From (1) and (3), we have
AD = CF ... (6)

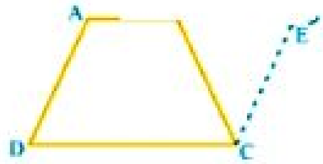
(iv) In ACFD,
AD = CF [∵ From (6)]
AD || CF [∵ From (5)]
Hence, ACFD is a parallelogram.

(v) In ACFD,
AC = DF [∵ ACFD is a parallelogram]

(vi) In ΔABC and ΔDEF,
AB = DE [∵ Given]
AC = DF [∵ Proved above]
BC = EF [∵ Given]
Hence, ΔABC ≅ ΔDEF [∵ SSS Congruency rule]

Question 12:

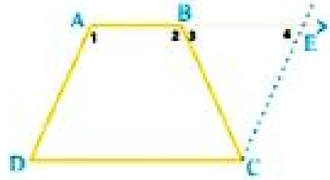
ABCD is a trapezium in which AB || CD and AD = BC (see Figure). Show that



(i) $\angle A = \angle B$
(ii) $\angle C = \angle D$
(iii) $\Delta ABC \cong \Delta BAD$
(iv) diagonal AC = diagonal BD
[***Hint***: Extend AB and draw a line through C parallel to DA intersecting AB produced at E.]

Answer 12:

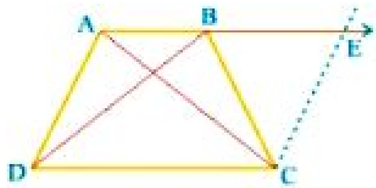
(i) Construction: Produce AB and draw a line through C parallel to AD, which intersects produced AB at E.



In AECD,
AE || DC [∵ Given]
AD || CE [∵ By construction]
Hence, AECD is a parallelogram.
AD = CE ... (1) [∵ Opposite sides of a parallelogram are equal]
AD = BC ... (2) [∵ Given]
Hence, CE = BC [∵ From the equation (1) and (2)]
Therefore, in ΔBCE,
$\angle 3 = \angle 4$ ... (3) [∵ In a triangle, the angles opposite to equal sides are equal]
Here, $\angle 2 + \angle 3 = 180°$ ... (4) [∵ Linear Pair]
$\angle 1 + \angle 4 = 180°$ ... (5) [∵ Co-interior angles]
Therefore, $\angle 2 + \angle 3 = \angle 1 + \angle 4$ [∵ From the equation (4) and (5)]
$\Rightarrow \angle 2 = \angle 1$ $\Rightarrow \angle B = \angle A$ [∵ $\angle 3 = \angle 4$]

(ii) ABCD is a trapezium in which AB || DC, hence,
$\angle 1 + \angle D = 180°$ ... (6) [∵ Co-interior angles]
$\angle 2 + \angle C = 180°$ ... (7) [∵ Co-interior angles]
Therefore, $\angle 1 + \angle D = \angle 2 + \angle C$ [∵ From the equation (6) and (7)]
$\Rightarrow \angle D = \angle C$ [∵ $\angle 2 = \angle 1$]

(iii) In ΔABC and ΔBAD,



BC = AD [∵ Given]
$\angle ABC = \angle BAD$ [∵ Proved above]
AB = AB [∵ Common]
Hence, $\Delta ABC \cong \Delta BAD$ [∵ SAS Congruency rule]

(iv) $\Delta ABC \cong \Delta BAD$ [∵ Proved above]
Diagonal AC = diagonal BD [∵ CPCT]

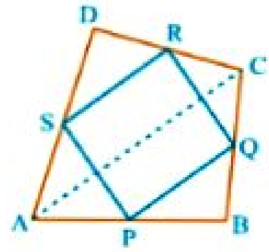
Exercise 8.2

Question 1:

ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see Figure). AC is a diagonal. Show that :

(i) SR || AC and SR = $\frac{1}{2}$AC **(ii)** PQ = SR

(iii) PQRS is a parallelogram.



Answer 1:

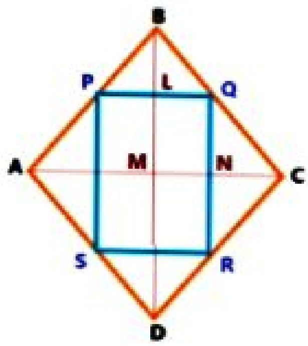
(i) In ΔACD,
S is mid-point of DA. [∵ Given]
R is mid-point of DC [∵ Given]
Hence, SR || AC and SR = $\frac{1}{2}$AC ... (1) [∵ Mid Point Theorem]

(ii) In ΔACD,
P is mid-point of AB [∵ Given]
Q is mid-point of BC [∵ Given]
Hence, PQ || AC and PQ = $\frac{1}{2}$AC ... (2) [∵ Mid Point Theorem]
From (1) and (2), we have
PQ || SR ... (3) [∵ PQ || AC and SR || AC]
and PQ = SR ... (4) [∵ SR = $\frac{1}{2}$AC and PQ = $\frac{1}{2}$AC]

(iii) In PQRS,
PQ || SR and PQ = SR [∵ From (3) and (4)]
Hence, PQRS is a parallelogram.

Question 2:

ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.



Answer 2:

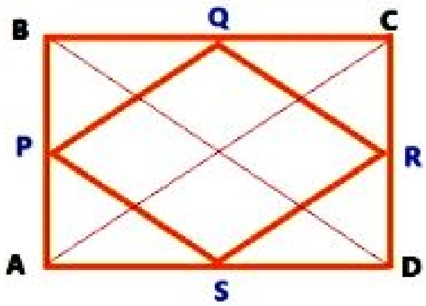
In ΔABC,
P is mid-point of AB [∵ Given]
Q is mid-point of BC [∵ Given]
Hence, PQ || AC and PQ = $\frac{1}{2}$AC ... (1) [∵ Mid Point Theorem]
Similarly, in ΔACD,
S is mid-point of AD [∵ Given]
R is mid-point of CD [∵ Given]
Hence, SR || AC and SR = $\frac{1}{2}$AC ... (2) [∵ Mid Point Theorem]
From (1) and (2), we have
PQ || SR ... (3) [∵ PQ || AC and SR || AC]
and PQ = SR ... (4) [∵ SR = $\frac{1}{2}$AC and PQ = $\frac{1}{2}$AC]
Hence, PQRS is a parallelogram.
Similarly, in ΔBCD,
Q is mid-point of BC [∵ Given]
R is mid-point of CD [∵ Given]

Hence, QR || BD [∵ Mid Point Theorem]
⇒ QN || LM ... (5)
and, LQ || MN ... (6) [∵ PQ || AC]
From (5) and (6), we have
LMNQ is a parallelogram.
Hence, ∠LMN = ∠LQN [∵ Opposite angles of a parallelogram]
But, ∠LMN = 90° [∵ Digonals of a rhombus are perpendicular to each other]
Hence, ∠LQN = 90°
A parallelogram whose one angle is right angle, is a rectangle. Hence, PQRS is a rectangle.

Question 3:

ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Answer 3:



In ΔABC,
P is mid-point of AB [∵ Given]
Q is mid-point of BC [∵ Given]
Hence, PQ || AC and PQ = $\frac{1}{2}$AC ... (1) [∵ Mid Point Theorem]

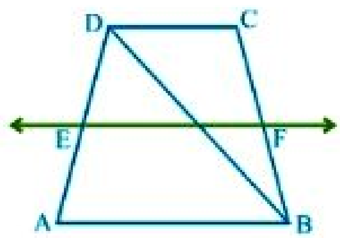
Similarly, in ΔACD,
S is mid-point of AD [∵ Given]
R is mid-point of CD [∵ Given]
Hence, SR || AC and SR = $\frac{1}{2}$AC ... (2) [∵ Mid Point Theorem]

From (1) and (2), we have
PQ || SR ... (3) [∵ PQ || AC and SR || AC]
and PQ = SR ... (4) [∵ SR = $\frac{1}{2}$AC and PQ = $\frac{1}{2}$AC]
Hence, PQRS is a parallelogram.

Similarly, in ΔBCD,
Q is mid-point of BC [∵ Given]
R is mid-point of CD [∵ Given]
Hence, QR = $\frac{1}{2}$BD ... (5) [∵ Mid Point Theorem]
Given that: AC = BD ... (6) [∵ Diagonals of a rectangle are equal]
From (1), (5) and (6), we have
PQ = QR
A parallelogram whose adjacent sides are equal, is a rhombus. Hence PQRS is a rhombus.

Question 4:

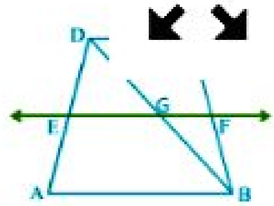
ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see Figure). Show that F is the mid-point of BC.



Answer 4:

In ΔABD,
E is mid-point of AD [∵ Given]
and EG || AB [∵ Given]
Hence, G is mid-point of BD [∵ Converse of Mid-Point Theorem]

Similarly,
In ΔBCD,



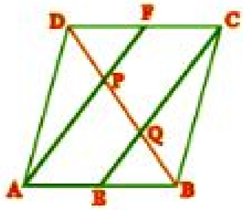
G is mid-point of BD	[∵ Proved above]
and FG || DC	[∵ Given]
Hence, F is mid-point of BC	[∵ Converse of Mid-Point Theorem]

Question 5:

In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see Figure). Show that the line segments AF and EC trisect the diagonal BD.

Answer 5:

In quadrilateral ABCD,
AB = CD [∵ Given]
$\frac{1}{2}AB = \frac{1}{2}CD$
⇒AE = CF [∵ E and F are the mid-points of AB and CD respectively]
In quadrilateral AECF,
AE = CF [∵ Proved above]
AE || CF [∵ Opposite sides of a parallelogram]
Hence, AECF is a parallelogram.



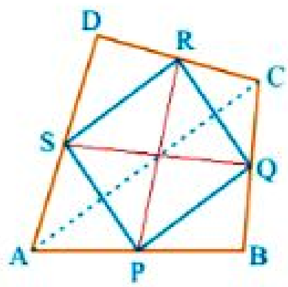
In ΔDCQ,
F is mid-point of DC [∵ Given]
and FP || CQ [∵ AECF is a parallelogram]
Hence, P is mid-point of DQ [∵ Converse of Mid-Point Theorem]
Hence, DP = PQ ... (1)
Similarly,
In ΔABP,
E is mid-point of AB [∵ Given]
and EQ || AP [∵ AECF is a parallelogram]
Hence, Q is mid-point of PB [∵ Converse of Mid-Point Theorem]
Hence, PQ = QB ... (2)
From (1) and (2), we have
DP = PQ = QB
Hence, line segment AF and EC trisect BD.

Question 6:

Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Answer 6:

Given: ABCD is a quadrilateral in which P, Q, R and S are the mid-points of AB, BC, CD and DA respectively.
In ΔACD,
S is mid-point of DA [∵ Given]
R is mid-point of DC [∵ Given]
Hence, SR || AC and $SR = \frac{1}{2}AC$ ... (1) [∵ Mid-Point Theorem]



In ΔACD,
P is mid-point of AB [∵ Given]
Q is mid-point of BC [∵ Given]
Hence, PQ || AC and $PQ = \frac{1}{2}AC$ ... (2) [∵Mid-Point Theorem]

From (1) and (2), we have

PQ || SR ... (3) [∵ PQ || AC and SR || AC]

and PQ = SR ... (4) [∵ SR = $\frac{1}{2}$AC and PQ = $\frac{1}{2}$AC]

In quadrilateral PQRS,

PQ || SR and PQ = SR [∵ From (3) and (4)]

Hence, PQRS is a parallelogram and diagonals of parallelogram bisects each other.

Therefore, SQ and PR bisects each other.

Question 7:

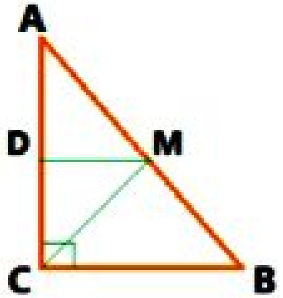
ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

(i) D is the mid-point of AC

(ii) MD ⊥ AC

(iii) CM = MA = $\frac{1}{2}$AB

Answer 7:



(i) In ΔABC,

M is mid-point of AB [∵ Given]

and DM || BC [∵ Given]

Hence, D is mid-point of AC [∵ Converse of Mid-Point Theorem]

(ii) ∠ADM = ∠ACB [∵ Corresponding Angles]

⇒∠ADM = 90° [∵ ∠ACB 90°]

Hence, MD ⊥ AC

(iii) In ΔAMD and ΔCMD,

AD = D C [∵ Proved above]

∠ADM = ∠CDM [∵ Each 90°]

DM = DM [∵ Common]

Hence, ΔAMD ≅ ΔCMD [∵ SAS Congruency rule]

AM = CM [∵ CPCT]

But AM = $\frac{1}{2}$ AB [∵ Given]

Therefore, CM = AM = $\frac{1}{2}$AB