Mathematics

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(Chapter - 1)(Number Systems)

(Class - 9)

Exercise 1.1

Question 1:

Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

Answer 1:

Yes, zero is a rational number. It can be written in the form of $\frac{p}{q}$. For example: $\frac{0}{1}, \frac{0}{2}, \frac{0}{5}$ are rational numbers, where p and q are integers and $q \neq 0$.

Question 2:

Find six rational numbers between 3 and 4.

Answer 2:

First Method: To get six rational number between 3 and 4, the denominator must be $6 + 1 = 7$.

Here, $3 = \frac{3 \times 7}{7} = \frac{21}{7}$ and $4 = \frac{4 \times 7}{7} = \frac{28}{7}$

So, the six rational can be obtained by changing numerator from 22 to 27.

Therefore, the rational numbers are: $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$

Second Method: six rational numbers between 3 and 4 are 3.1, 3.2, 3.3, 3.4, 3.5 and 3.6

Question 3:

Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Answer 3:

By converting these numbers into decimal, we have

$\frac{3}{5} = 0.6$ and $\frac{4}{5} = 0.8$

Hence, five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$ are 0.61, 0.62, 0.63, 0.64 and 0.65.

Question 4:

State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

Answer 4:

(i) True, as whole number is the collection of Natural numbers and 0.

(ii) False, because negative integers are not whole numbers.

(iii) False, rational numbers like $\frac{3}{5}, \frac{2}{3}, \frac{7}{9}$ are not the whole numbers.

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Exercise 1.2

Question 1:

State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

(iii) Every real number is an irrational number.

Answer 1:

(i) True, as the collection of all rational and irrational number is real numbers.

(ii) False, there are infinite number on number line between $\sqrt{2}$ and $\sqrt{3}$ that can't be represented as $\sqrt{m}$, m being a natural number.

(iii) False, because real numbers can be rational also.

Question 2:

Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Answer 2:

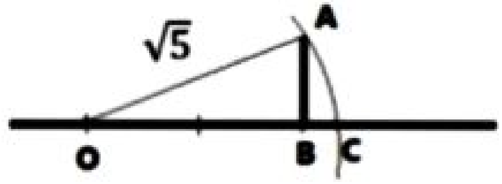
The square roots of all positive integers are not irrational, for example $\sqrt{4} = 2$, which is a rational number.

Question 3:

Show how $\sqrt{5}$ can be represented on the number line.

Answer 3:

To represent $\sqrt{5}$ on number line, take OB = 2 units and make a perpendicular AB at B such that AB = 1 unit.



Now by Pythagoras theorem, the length of OA is $\sqrt{5}$. Now taking O as centre and OA as radius, mark an arc on OB, which intersects at C. Hence, OC = $\sqrt{5}$.

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Exercise 1.3

Question 1:

Write the following in decimal form and say what kind of decimal expansion each has:

(i) $\frac{36}{100}$ (ii) $\frac{1}{11}$ (iii) $4\frac{1}{8}$

(iv) $\frac{3}{13}$ (v) $\frac{2}{11}$ (vi) $\frac{329}{400}$

Answer 1:

(i) $\frac{36}{100} = 0.3$, Terminating.

(ii) $\frac{1}{11} = 0.\overline{09}$, Recurring & Non-terminating.

(iii) $4\frac{1}{8} = 4.125$, Terminating.

(iv) $\frac{3}{13} = 0.\overline{230769}$, Recurring & Non-terminating.

(v) $\frac{2}{11} = 0.\overline{18}$, Recurring & Non-terminating.

(vi) $\frac{329}{400} = 0.8225$, Terminating.

Question 2:

You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division? If so, how?

[Hint: Study the remainders while finding the value of $\frac{1}{7}$ carefully.]

Answer 2:

Without actual long division, the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are as follows:

$\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0.\overline{142857} = 0.\overline{285714}$

$\frac{3}{7} = 3 \times \frac{1}{7} = 3 \times 0.\overline{142857} = 0.\overline{428571}$

$\frac{4}{7} = 4 \times \frac{1}{7} = 4 \times 0.\overline{142857} = 0.\overline{571428}$

$\frac{5}{7} = 5 \times \frac{1}{7} = 5 \times 0.\overline{142857} = 0.\overline{714285}$

$\frac{6}{7} = 6 \times \frac{1}{7} = 6 \times 0.\overline{142857} = 0.\overline{857142}$

Question 3:

Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

(i) $0.\bar{6}$ (ii) $0.4\bar{7}$ (iii) $0.\overline{001}$

Answer 3:

(i) $0.\bar{6}$

Let $x = 0.\bar{6} \Rightarrow x = 0.6666\ldots$ ... (i)

Multiplying equation (i) by 10 both sides

$10x = 6.6666\ldots$

$\Rightarrow 10x = 6 + 0.6666\ldots$

$\Rightarrow 10x = 6 + x$ [From equation (i)]

$\Rightarrow 10x - x = 6 \Rightarrow 9x = 6 \Rightarrow x = \frac{6}{9} = \frac{2}{3}$

(ii) $0.4\bar{7}$

Let $x = 0.4\bar{7}$

$\Rightarrow x = 0.47777 \ldots$ ... (i)

Multiplying equation (i) by 10 both sides

$\Rightarrow 10x = 4.7777 \ldots$ ... (ii)

Multiplying equation (ii) by 10 both sides

$100x = 47.7777 \ldots$

$\Rightarrow 100x = 43 + 4.7777 \ldots$

$\Rightarrow 100x = 43 + 10x$ [From equation (ii)]

$\Rightarrow 100x - 10x = 43$

$\Rightarrow 90x = 43$

$\Rightarrow x = \frac{43}{90}$

(iii) $0.\overline{001}$

Let $x = 0.\overline{001}$

$\Rightarrow x = 0.001001001 \ldots$ ... (i)

Multiplying equation (i) by 1000 both sides

$1000x = 1.001001001 \ldots$

$\Rightarrow 1000x = 1 + 0.001001001 \ldots$

$\Rightarrow 1000x = 1 + x$ [From equation (i)]

$\Rightarrow 1000x - x = 1$

$\Rightarrow 999x = 1$

$\Rightarrow x = \frac{1}{999}$

Question 4:

Express 0.99999 ... in the form of $\frac{p}{q}$. Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

Answer 4:

0.99999 ...

Let $x = 0.99999 \ldots$ ... (i)

Multiplying equation (i) by 10 both sides

$10x = 9.99999 \ldots$

$\Rightarrow 10x = 9 + 0.99999 \ldots \ldots$

$\Rightarrow 10x = 9 + x$ [From equation (i)]

$\Rightarrow 10x - x = 9 \Rightarrow 9x = 9 \Rightarrow x = \frac{9}{9} = 1$

The answer makes sense as 0.99999 ... is very close to 1, that is why we can say that 0.99999 = 1.

Question 5:

What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer?

Answer 5:

The maximum number of digits that can be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$ is 16 (less than 17).

By performing the actual division, we get

$$\frac{1}{17} = 0.\overline{0588235294117647}$$

So, the maximum number of digits that can be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$ is 16.

Question 6:

Look at several examples of rational numbers in the form $\frac{p}{q}$ $(q \neq 0)$, where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Answer 6:

$\frac{2}{5} = 0.4$, $\frac{1}{10} = 0.1$, $\frac{3}{2} = 1.5$, $\frac{7}{8} = 0.875$

The denominator of all the rational numbers are in the form of $2^m \times 5^n$, where m and n are integers.

Question 7:

Write three numbers whose decimal expansions are non-terminating non-recurring.

Answer 7:

Three non-terminating non-recurring decimals:

1) 0.414114111411114 ...
2) 2.01001000100001 ...
3) $\pi = 3.1416 ...$

Question 8:

Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Answer 8:

$$\frac{5}{7} = 0.\overline{714285} \text{ and } \frac{9}{11} = 0.\overline{81}$$

We know that there are infinite many irrational numbers between two rational numbers. So the three irrational numbers are:

1) 0.72722722272222 ...
2) 0.73733733373333 ...
3) 0.74744744474444 ...

Question 9:

Classify the following numbers as rational or irrational:

(i) $\sqrt{23}$ (ii) $\sqrt{225}$ (iii) 0.3796
(iv) 7.478478 ... (v) 1.101001000100001 ...

Answer 9:

(i) $\sqrt{23}$, Irrational number
(ii) $\sqrt{225} = 15$, Rational number
(iii) 0.3796, Rational number
(iv) 7.478478 ... $= 7.\overline{478}$, Rational number
(v) 1.101001000100001 ..., Irrational number

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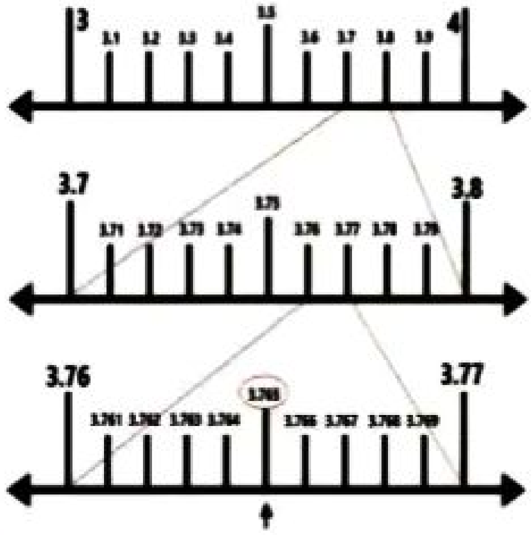
Exercise 1.4

Question 1:

Visualise 3.765 on the number line, using successive magnification.

Answer 1:

- First of all, we observe that 3.765 lies between 3 and 4. Divide this portion into 10 equal parts.
- In the next step, we locate 3.765 between 3.7 and 3.8.
- To get a more accurate visualisation of representation, we divide this portion of number line into 10 equal parts and use a magnifying glass to visualize that 3.765 lies between 3.76 and 3.77.
- Now to visualise 3.765 still more accurately, we divide the portion between 3.76 and 3.77 into 10 equal parts and locate 3.765.

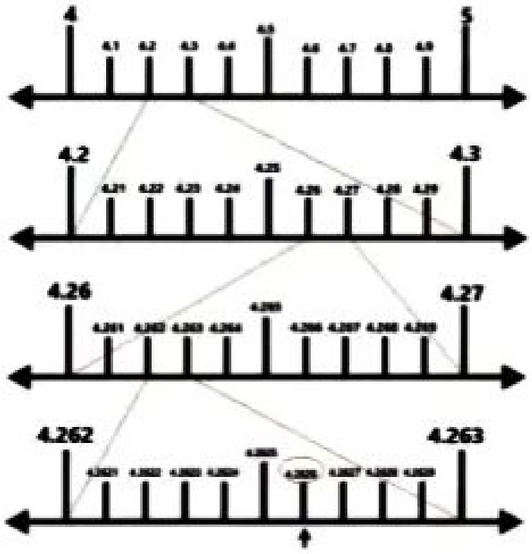


Question 2:

Visualise $4.\overline{26}$ on the number line, up to 4 decimal places.

Answer 2:

- First of all, we observe that 4.2626 ($4.\overline{26}$) lies between 4 and 5. Divide this portion into 10 equal parts.
- In the next step, we locate 4.2626 between 4.2 and 4.3.
- To get a more accurate visualisation of representation, we divide this portion of number line into 10 equal parts and use a magnifying glass to visualize that 4.2626 lies between 4.262 and 4.263.
- Now to visualise 4.2626 still more accurately, we divide the portion between 4.262 and 4.263 into 10 equal parts and locate 4.2626.



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Exercise 1.5

Question 1:

Classify the following numbers as rational or irrational:

(i) $2 - \sqrt{5}$ (ii) $(3 - \sqrt{23}) - \sqrt{23}$ (iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$ (iv) $\frac{1}{\sqrt{2}}$ (v) 2π

Answer 1:

(i) $2 - \sqrt{5}$ Irrational number.
(ii) $(3 - \sqrt{23}) - \sqrt{23} = 3$ Rational number.
(iii) $\frac{2\sqrt{7}}{7\sqrt{7}} = \frac{2}{7}$ Rational number.
(iv) $\frac{1}{\sqrt{2}}$ Irrational number.
(v) 2π Irrational number.

Question 2:

Simplify each of the following expressions:

(i) $(3 + \sqrt{3})(2 + \sqrt{2})$ (ii) $(3 + \sqrt{3})(3 - \sqrt{3})$ (iii) $(\sqrt{5} + \sqrt{2})^2$ (iv) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$

Answer 2:

(i) $(3 + \sqrt{3})(2 + \sqrt{2}) = 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$
(ii) $(3 + \sqrt{3})(3 - \sqrt{3}) = 3^2 - (\sqrt{3})^2 = 9 - 3 = 6$ $[\because (a + b)(a - b) = a^2 - b^2]$
(iii) $(\sqrt{5} + \sqrt{2})^2 = (\sqrt{5})^2 + (\sqrt{2})^2 + 2 \times \sqrt{5} \times \sqrt{2} = 7 + 2\sqrt{10}$ $[\because (a + b)^2 = a^2 + b^2 + 2ab]$
(iv) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2}) = (\sqrt{5})^2 - (\sqrt{2})^2 = 5 - 2 = 3$ $[\because (a - b)(a + b) = a^2 - b^2]$

Question 3:

Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is $\pi = \frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Answer 3:

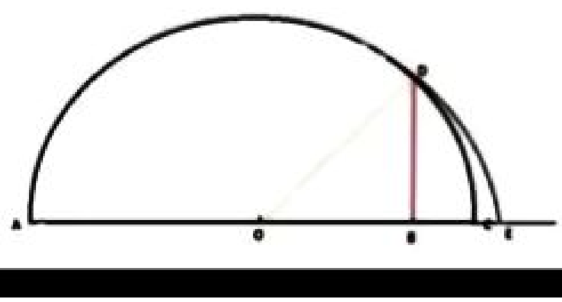
With a scale or tape we get only an approximate rational number as the result of our measurement. That is why π can be approximately represented as a quotient of two rational numbers. As a matter of mathematical truth it is irrational.

Question 4:

Represent $\sqrt{9.3}$ on the number line.

Answer 4:

To represent $\sqrt{9.3}$ on the number line, draw AB = 9.3 units. Now produce AB to C, such that BC = 1 unit. Draw the perpendicular bisector of AC which intersects AC at O. Taking O as centre and OA as radius, draw a semi-circle which intersects at D to the perpendicular at B. Now taking O as centre and OD as radius, draw an arc, which intersects AC produced at E. Hence, OE = $\sqrt{9.3}$.



Question 5:
Rationalise the denominators of the following:

(i) $\frac{1}{\sqrt{7}}$ (ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$ (iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$ (iv) $\frac{1}{\sqrt{7}-2}$

Answer 5:
(i) $\frac{1}{\sqrt{7}}$

$$=\frac{1}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}=\frac{\sqrt{7}}{7}$$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

$$=\frac{1}{\sqrt{7}-\sqrt{6}}\times\frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}$$

$$=\frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2}$$

$$=\frac{\sqrt{7}+\sqrt{6}}{7-6}$$

$$=\sqrt{7}+\sqrt{6}$$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

$$=\frac{1}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$=\frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5})^2-(\sqrt{2})^2}$$

$$=\frac{\sqrt{5}-\sqrt{2}}{5-2}$$

$$=\frac{\sqrt{5}-\sqrt{2}}{3}$$

(iv) $\frac{1}{\sqrt{7}-2}$

$$=\frac{1}{\sqrt{7}-2}\times\frac{\sqrt{7}+2}{\sqrt{7}+2}$$

$$=\frac{\sqrt{7}+2}{(\sqrt{7})^2-(2)^2}$$

$$=\frac{\sqrt{7}+2}{7-4}=\frac{\sqrt{7}+2}{3}$$

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Exercise 1.6

Question 1:

Find:

(i) $64^{\frac{1}{2}}$ (ii) $32^{\frac{1}{5}}$ (iii) $125^{\frac{1}{3}}$

Answer 1:

(i) $64^{\frac{1}{2}} = (8^2)^{\frac{1}{2}} = 8^{2\times\frac{1}{2}} = 8$

(ii) $32^{\frac{1}{5}} = (2^5)^{\frac{1}{5}} = 2^{5\times\frac{1}{5}} = 2$

(iii) $125^{\frac{1}{3}} = (5^3)^{\frac{1}{3}} = 5^{3\times\frac{1}{3}} = 5$

Question 2:

Find:

(i) $9^{\frac{3}{2}}$ (ii) $32^{\frac{2}{5}}$ (iii) $16^{\frac{3}{4}}$ (iv) $125^{\frac{-1}{3}}$

Answer 2:

(i) $9^{\frac{3}{2}} = (3^2)^{\frac{3}{2}} = 3^{2\times\frac{3}{2}} = 3^3 = 9$

(ii) $32^{\frac{2}{5}} = (2^5)^{\frac{2}{5}} = 2^{5\times\frac{2}{5}} = 2^2 = 4$

(iii) $16^{\frac{3}{4}} = (2^4)^{\frac{3}{2}} = 2^{4\times\frac{3}{4}} = 2^3 = 8$

(iv) $125^{-\frac{1}{3}} = (5^3)^{-\frac{1}{3}} = 5^{3\times-\frac{1}{3}} = 5^{-1} = \frac{1}{5} = 5$

Question 3:

Simplify:

(i) $2^{\frac{2}{3}}.2^{\frac{1}{5}}$ (ii) $\left(\frac{1}{3^3}\right)^7$ (iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$ (iv) $7^{\frac{1}{2}}.8^{\frac{1}{2}}$

Answer 3:

(i) $2^{\frac{2}{3}}.2^{\frac{1}{5}} = 2^{\frac{2}{3}+\frac{1}{5}} = 2^{\frac{10+3}{15}} = 2^{\frac{13}{15}}$

(ii) $\left(\frac{1}{3^3}\right)^7 = (3^{-3})^7 = 3^{-21}$

(iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}} \times 11^{-\frac{1}{4}} = 11^{\frac{1}{2}-\frac{1}{4}} = 11^{\frac{2-1}{4}} = 11^{\frac{1}{4}}$

(iv) $7^{\frac{1}{2}}.8^{\frac{1}{2}} = (7\times 8)^{\frac{1}{2}} = 56^{\frac{1}{2}}$