Mathematics

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(Chapter - 2)(Polynomials)

(Class - 9)

Exercise 2.1

Question 1:

Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 - 3x + 7$ (ii) $y^2 + \sqrt{2}$ (iii) $3\sqrt{t} + t\sqrt{2}$ (iv) $y + \frac{2}{y}$ (v) $x^{10} + y^3 + t^{50}$

Answer 1:

(i) $4x^2 - 3x + 7$ Polynomials in one variable as it contains only one variable x.

(ii) $y^2 + \sqrt{2}$ Polynomials in one variable as it contains only one variable y.

(iii) $3\sqrt{t} + t\sqrt{2} = 3t^{\frac{1}{2}} + t\sqrt{2}$, It is in one variable but not a polynomial as it contains $(t^{\frac{1}{2}})$, in which power is not a whole number.

(iv) $y + \frac{2}{y} = y + 2y^{-1}$, It is in one variable but not a polynomial as it contains (y^{-1}), in which power is not a whole number.

(v) $x^{10} + y^3 + t^{50}$, It is a polynomials in three variable as it contains three variable $(x,\ y,\ t)$.

Question 2:

Write the coefficients of x^2 in each of the following:

(i) $2 + x^2 + x$ (ii) $2 - x^2 + x^3$ (iii) $\frac{\pi}{2}x^2 + x$ (iv) $\sqrt{2}x - 1$

Answer 2:

(i) In $2 + x^2 + x$ the coefficient of x^2 is 1.

(ii) In $2 - x^2 + x^3$ the coefficient of x^2 is -1.

(iii) In $\frac{\pi}{2}x^2 + x$ the coefficient of x^2 is $\frac{\pi}{2}$.

(iv) In $\sqrt{2}x - 1 = 0.x^2 + \sqrt{2}x - 1$ the coefficient of x^2 is 0.

Question 3:

Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Answer 3:

A binomial of degree 35 = $x^{35} + 3$ and a monomial of degree 100 = $3x^{100}$

Question 4:

Write the degree of each of the following polynomials:

(i) $5x^3 + 4x^2 + 7x$ (ii) $4-y^2$ (iii) $5t - \sqrt{7}$ (iv) 3

Answer 4:

(i) The degree of $5x^3 + 4x^2 + 7x$ is 3.

(ii) The degree of $4-y^2$ is 2.

(iii) The degree of $5t - \sqrt{7} = 5t^1 - \sqrt{7}$ is 1.

(iv) The degree of $3 = 3x^0$ is 0.

Question 5:

Classify the following as linear, quadratic and cubic polynomials:

(i) $x^2 + x$ (ii) $x - x^3$ (iii) $y + y^2 + 4$ (iv) $1 + x$

(v) $3t$ (vi) r^2 (vii) $7x^3$

Answer 5:

(i) $x^2 + x$ Quadratic polynomial.

(ii) $x - x^3$ Cubic polynomial.

(iii) $y + y^2 + 4$ Quadratic polynomial.

(iv) $1 + x$ Linear polynomial.

(v) $3t$ Linear polynomial.

(vi) r^2 Quadratic polynomial.

(vii) $7x^3$ Cubic polynomial.

Exercise 2.2

Question 1:

Find the value of the polynomial $5x - 4x^2 + 3$ at:

(i) $x = 0$ (ii) $x = -1$ (iii) $x = 2$

Answer 1:

Let $p(x) = 5x - 4x^2 + 3$

(i) Putting $x = 0$, we get: $p(0) = 5 \times 0 - 4(0)^2 + 3 = 3$

(ii) Putting $x = -1$, we get: $p(-1) = 5 \times (-1) - 4(-1)^2 + 3 = -5 - 4 + 3 = -6$

(iii) Putting $x = 2$, we get: $p(2) = 5 \times 2 - 4(2)^2 + 3 = 10 - 16 + 3 = -3$

Question 2:

Find $p(0)$, $p(1)$ and $p(2)$ for each of the following polynomials:

(i) $p(y) = y^2 - y + 1$ (ii) $p(t) = 2 + t + 2t^2 - t^3$

(iii) $p(x) = x^3$ (iv) $p(x) = (x - 1)(x + 1)$

Answer 2:

(i) $p(y) = y^2 - y + 1$

Putting $y = 0$, we get

$p(0) = 0^2 - 0 + 1 = 1$

Putting $y = 1$, we get

$p(1) = 1^2 - 1 + 1 = 1$

Putting $y = 2$, we get

$p(2) = 2^2 - 2 + 1 = 3$

(ii) $p(t) = 2 + t + 2t^2 - t^3$

Putting $t = 0$, we get

$p(0) = 2 + 0 + 2(0)^2 - (0)^3 = 2$

Putting $t = 1$, we get

$p(1) = 2 + 1 + 2(1)^2 - (1)^3 = 2 + 1 + 2 - 1 = 4$

Putting $t = 2$, we get

$p(2) = 2 + 2 + 2(2)^2 - (2)^3 = 2 + 2 + 8 - 8 = 4$

(iii) $p(x) = x^3$

Putting $x = 0$, we get

$p(0) = (0)^3 = 0$

Putting $x = 1$, we get

$p(1) = (1)^3 = 1$

Putting $x = 2$, we get

$p(2) = (2)^3 = 8$

(iv) $p(x) = (x - 1)(x + 1)$

Putting $x = 0$, we get

$p(0) = (0 - 1)(0 + 1) = -1$

Putting $x = 1$, we get

$p(1) = (1 - 1)(1 + 1) = 0 \times 2 = 0$

Putting $x = 2$, we get

$p(2) = (2 - 1)(2 + 1) = 3$

Question 3:

Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x) = 3x + 1;\quad x = -\frac{1}{3}$

(ii) $p(x) = 5x - \pi;\quad x = \frac{4}{5}$

(iii) $p(x) = x^2 - 1;\quad x = 1, -1$

(iv) $p(x) = (x + 1)(x - 2);\quad x = -1, 2$

(v) $p(x) = x^2;\quad x = 0$

(vi) $p(x) = lx + m;\quad x = -\frac{m}{l}$

(vii) $p(x) = 3x^2 - 1;\quad x = -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

(viii) $p(x) = 2x + 1;\quad x = \frac{1}{2}$

Answer 3:

(i) $p(x) = 3x + 1;\quad x = -\frac{1}{3}$

Putting $x = -\frac{1}{3}$, we get

$p\left(-\frac{1}{3}\right) = 3 \times \left(-\frac{1}{3}\right) + 1 = -1 + 1 = 0$

Here, $p\left(-\frac{1}{3}\right) = 0$, Hence, $x = -\frac{1}{3}$ is a solution of $p(x) = 3x + 1$.

(ii) $p(x) = 5x - \pi;\quad x = \frac{4}{5}$

Putting $x = \frac{4}{5}$, we get

$p\left(\frac{4}{5}\right) = 5 \times \left(\frac{4}{5}\right) - \pi = 4 - \pi$

Here, $p\left(\frac{4}{5}\right) \neq 0$, Hence, $x = \frac{4}{5}$ is not a solution of $p(x) = 5x - \pi$.

(iii) $p(x) = x^2 - 1;\quad x = 1, -1$

Putting $x = 1$, we get

$p(1) = (1)^2 - 1 = 1 - 1 = 0$

Here, $p(1) = 0$, Hence, $x = 1$ is a solution of $p(x) = x^2 - 1$.

Putting $x = -1$, we get

$p(-1) = (-1)^2 - 1 = 1 - 1 = 0$

Here, $p(-1) = 0$, Hence, $x = -1$ is a solution of $p(x) = x^2 - 1$.

(iv) $p(x) = (x + 1)(x - 2);\quad x = -1, 2$

Putting $x = -1$, we get

$p(-1) = (-1 + 1)(-1 - 2) = 0 \times (-3) = 0$

Here, $p(-1) = 0$, Hence, $x = -1$ is a solution of $p(x) = (x + 1)(x - 2)$.

Putting $x = 2$, we get

$p(2) = (2 + 1)(2 - 2) = 3 \times 9 = 0$

Here, $p(2) = 0$, Hence, $x = 2$ is a solution of $p(x) = (x + 1)(x - 2)$.

(v) $p(x) = x^2;\quad x = 0$

Putting $x = 0$, we get

$p(0) = (0)^2 = 0$

Here, $p(0) = 0$, Hence, $x = 0$ is a solution of $p(x) = x^2$.

(vi) $p(x) = lx + m;\quad x = -\frac{m}{l}$

Putting $x = -\frac{m}{l}$, we get

$p\left(-\frac{m}{l}\right) = l \times \left(-\frac{m}{l}\right) + m = -m + m = 0$

Here, $p\left(-\frac{m}{l}\right) = 0$,

Hence, $x = -\frac{m}{l}$ is a solution of $p(x) = lx + m$.

$= (-1)^3 + (-1)^2 + (-1) + 1 = -1 + 1 - 1 + 1 = 0$
Since, remainder $p(-1) = 0$, hence $g(x)$ is a factor of $p(x)$.

(ii) $p(x) = x^3 + 3x^2 + 3x + 1$ and $g(x) = x + 2$
Putting $x + 2 = 0$, we get, $x = -2$
Using remainder theorem, when $p(x) = x^3 + 3x^2 + 3x + 1$ is divided by $g(x) = x + 2$, remainder is given by $p(-2)$
$= (-2)^3 + 3(-2)^2 + 3(-2) + 1 = -8 + 12 - 6 + 1 = -1$
Since, remainder $p(-2) \neq 0$, hence $g(x)$ is not a factor of $p(x)$.

(iii) $p(x) = x^3 - 4x^2 + x + 6$ and $g(x) = x - 3$
Putting $x - 3 = 0$, we get, $x = 3$
Using remainder theorem, when $p(x) = x^3 - 4x^2 + x + 6$ is divided by $g(x) = x - 3$, remainder is given by $p(3)$
$= (3)^3 - 4(3)^2 + (3) + 6 = 27 - 36 + 3 + 6 = 0$
Since, remainder $p(3) = 0$, hence $g(x)$ is a factor of $p(x)$.

Question 3:

Find the value of k, if $x - 1$ is a factor of $p(x)$ in each of the following cases:
(i) $p(x) = x^2 + x + k$
(ii) $p(x) = 2x^2 + kx + \sqrt{2}$
(iii) $p(x) = kx^2 - \sqrt{2}x + 1$
(iv) $p(x) = kx^2 - 3x + k$

Answer 3:

(i) $p(x) = x^2 + x + k$
Putting $x - 1 = 0$, we get, $x = 1$
Using remainder theorem, when $p(x) = x^2 + x + k$ is divided by $x - 1$, remainder is given by $p(1)$
$= (1)^2 + (1) + k = 2 + k$
Since $x - 1$ is a factor of $p(x)$, hence remainder $p(1) = 0 \Rightarrow 2 + k = 0 \Rightarrow k = -2$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$
Putting $x - 1 = 0$, we get, $x = 1$
Using remainder theorem, when $p(x) = 2x^2 + kx + \sqrt{2}$ is divided by $x - 1$, remainder is given by
$p(1) = 2(1)^2 + k(1) + \sqrt{2} = 2 + k + \sqrt{2}$
Since $x - 1$ is a factor of $p(x)$, hence remainder $p(1) = 0 \Rightarrow 2 + k + \sqrt{2} = 0 \Rightarrow k = -2 - \sqrt{2}$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$
Putting $x - 1 = 0$, we get, $x = 1$

Using remainder theorem, when $p(x) = kx^2 - \sqrt{2}x + 1$ is divided by $x - 1$, remainder is given by $p(1)$
$= k(1)^2 - \sqrt{2}(1) + 1 = k - \sqrt{2} + 1$
Since $x - 1$ is a factor of $p(x)$, hence remainder $p(1) = 0 \Rightarrow k - \sqrt{2} + 1 = 0 \Rightarrow k = \sqrt{2} - 1$

(iv) $p(x) = kx^2 - 3x + k$
Putting $x - 1 = 0$, we get, $x = 1$

Using remainder theorem, when $p(x) = kx^2 - 3x + k$ is divided by $x - 1$, remainder is given by $p(1) = k(1)^2 - 3(1) + k = 2k - 3$

Since $x - 1$ is a factor of $p(x)$, hence remainder $p(1) = 0 \Rightarrow 2k - 3 = 0 \Rightarrow k = \frac{3}{2}$

Question 4:

Factorise:

(i) $12x^2 - 7x + 1$
(ii) $2x^2 + 7x + 3$
(iii) $6x^2 + 5x - 6$
(iv) $3x^2 - x - 4$

Answer 4:

(i) $12x^2 - 7x + 1$
$= 12x^2 - (4 + 3)x + 1$
$= 12x^2 - 4x - 3x + 1$
$= 4x(3x - 1) - 1(3x - 1)$
$= (3x - 1)(4x - 1)$

(ii) $2x^2 + 7x + 3$
$= 2x^2 + (6 + 1)x + 3$
$= 2x^2 + 6x + x + 3$
$= 2x(x + 3) + 1(x + 3)$
$= (x + 3)(2x + 1)$

(iii) $6x^2 + 5x - 6$
$= 6x^2 + (9 - 4)x - 6$
$= 6x^2 + 9x - 4x - 6$
$= 3x(2x + 3) - 2(2x + 3)$
$= (2x + 3)(3x - 2)$

(iv) $3x^2 - x - 4$
$= 3x^2 - (4 - 3)x - 4$
$= 3x^2 - 4x + 3x - 4$
$= x(3x - 4) + 1(3x - 4)$
$= (3x - 4)(x + 1)$

Question 5:

Factorise:

(i) $x^3 - 2x^2 - x + 2$
(ii) $x^3 - 3x^2 - 9x - 5$
(iii) $x^3 + 13x^2 + 32x + 20$
(iv) $2y^3 + y^2 - 2y - 1$

Answer 5:

(i) $x^3 - 2x^2 - x + 2$

Let $p(x) = x^3 - 2x^2 - x + 2$

Putting $x = 1$, we get

$p(1) = (1)^3 - 2(1)^2 - (1) + 2 = 1 - 2 - 1 + 2 = 0$

$\Rightarrow x - 1$ is a factor of $p(x)$.

$$\begin{array}{r|l} & x^2 - x - 2 \\ \hline x - 1 & x^3 - 2x^2 - x + 2 \\ & x^3 - x^2 \\ & - \quad + \\ \hline & \quad -x^2 - x + 2 \\ & \quad -x^2 + x \\ & \quad + \quad - \\ \hline & \qquad -2x + 2 \\ & \qquad -2x + 2 \\ & \qquad + \quad - \\ \hline & \qquad\quad 0 \end{array}$$

So, $p(x) = (x - 1)(x^2 - x - 2)$
$= (x - 1)(x^2 - x - 2) = (x - 1)[x^2 - (2 - 1)x - 2]$
$= (x - 1)[x^2 - 2x + x - 2] = (x - 1)[x(x - 2) + 1(x - 2)]$
$= (x - 1)(x - 2)(x + 1)$

(iv) $2y^3 + y^2 - 2y - 1$
Let $p(y) = y^3 + y^2 - 2y - 1$
Putting $y = -1$, we get
$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1 = -2 + 1 + 2 - 1 = 0$
$\Rightarrow y + 1$ is a factor of $p(y)$.

$$
\begin{array}{r|l}
 & 2y^2 - y - 1 \\ \hline
y + 1 & 2y^3 + y^2 - 2y - 1 \\
 & 2y^3 + 2y^2 \\
 & - \quad - \\ \hline
 & \quad -y^2 - 2y - 1 \\
 & \quad -y^2 - y \\
 & \quad + \quad + \\ \hline
 & \qquad\qquad -y - 1 \\
 & \qquad\qquad -y - 1 \\
 & \qquad\qquad + \quad + \\ \hline
 & \qquad\qquad 0
\end{array}
$$

So, $p(y) = (y + 1)(2y^2 - y - 1)$
$= (y + 1)(2y^2 - y - 1)$
$= (y + 1)[2y^2 - (2 - 1)y - 1]$
$= (y + 1)[2y^2 - 2y + y - 1]$
$= (y + 1)[2y(y - 1) + 1(y - 1)]$
$= (y + 1)(y - 1)(2y + 1)$

(vii) $p(x) = 3x^2 - 1;\ x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

Putting $x = -\frac{1}{\sqrt{3}}$, we get

$p\left(-\frac{1}{\sqrt{3}}\right) = 3\left(-\frac{1}{\sqrt{3}}\right)^2 - 1 = 3 \times \frac{1}{3} - 1 = 1 - 1 = 0$

Here, $p\left(-\frac{1}{\sqrt{3}}\right) = 0$, Hence, $x = -\frac{1}{\sqrt{3}}$ is a solution of $p(x) = 3x^2 - 1$.

Putting $x = \frac{2}{\sqrt{3}}$, we get

$p\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1 = 3 \times \frac{4}{3} - 1 = 4 - 1 = 3$

Here, $p\left(\frac{2}{\sqrt{3}}\right) \neq 0$, Hence, $x = \frac{2}{\sqrt{3}}$ is not a solution of $p(x) = 3x^2 - 1$.

(viii) $p(x) = 2x + 1;\ x = \frac{1}{2}$

Putting $x = \frac{1}{2}$, we get: $p\left(\frac{1}{2}\right) = 2 \times \left(\frac{1}{2}\right) + 1 = 1 + 1 = 2$

Here, $p\left(\frac{1}{2}\right) \neq 0$, Hence, $x = \frac{1}{2}$ is not a solution of $p(x) = 2x + 1$.

Question 4:

Find the zero of the polynomial in each of the following cases:

(i) $p(x) = x + 5$ **(ii)** $p(x) = x - 5$ **(iii)** $p(x) = 2x + 5$ **(iv)** $p(x) = 3x - 2$
(v) $p(x) = 3x$ **(vi)** $p(x) = ax,\ a \neq 0$ **(vii)** $p(x) = cx + d;\ c \neq 0,\ c,\ d$ are real numbers.

Answer 4:

(i) $p(x) = x + 5$
Putting $p(x) = 0$, we get: $x + 5 = 0 \Rightarrow x = -5$
Hence, $x = -5$ is a zero of the polynomial $p(x)$.

(ii) $p(x) = x - 5$
Putting $p(x) = 0$, we get: $x - 5 = 0 \Rightarrow x = 5$
Hence, $x = 5$ is a zero of the polynomial $p(x)$.

(iii) $p(x) = 2x + 5$
Putting $p(x) = 0$, we get: $2x + 5 = 0 \Rightarrow x = -\frac{5}{2}$
Hence, $x = -\frac{5}{2}$ is a zero of the polynomial $p(x)$.

(iv) $p(x) = 3x - 2$
Putting $p(x) = 0$, we get: $3x - 2 = 0 \Rightarrow x = \frac{2}{3}$
Hence, $x = \frac{2}{3}$ is a zero of the polynomial $p(x)$.

(v) $p(x) = 3x$
Putting $p(x) = 0$, we get: $3x = 0 \Rightarrow x = 0$
Hence, $x = 0$ is a zero of the polynomial $p(x)$.

(vi) $p(x) = ax,\ a \neq 0$
Putting $p(x) = 0$, we get: $ax = 0 \Rightarrow x = 0$
Hence, $x = 0$ is a zero of the polynomial $p(x)$.

(vii) $p(x) = cx + d;\ c \neq 0,\ c,\ d$ are real numbers.
Putting $p(x) = 0$, we get: $cx + d = 0 \Rightarrow x = -\frac{d}{c}$
Hence, $x = -\frac{d}{c}$ is a zero of the polynomial $p(x)$.

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Exercise 2.3

Question 1:

Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by:

(i) $x + 1$ **(ii)** $x - \frac{1}{2}$ **(iii)** x **(iv)** $x + \pi$ **(v)** $5 + 2x$

Answer 1:

Let $p(x) = x^3 + 3x^2 + 3x + 1$

(i) $x + 1$

Putting $x + 1 = 0$, we get, $x = -1$

Using remainder theorem, when $p(x) = x^3 + 3x^2 + 3x + 1$ is divided by $x + 1$, remainder is given by $p(-1)$

$= (-1)^3 + 3(-1)^2 + 3(-1) + 1$

$= -1 + 3 - 3 + 1$

$= 0$

(ii) $x - \frac{1}{2}$

Putting $x - \frac{1}{2} = 0$, we get, $x = \frac{1}{2}$

Using remainder theorem, when $p(x) = x^3 + 3x^2 + 3x + 1$ is divided by $x - \frac{1}{2}$, remainder is given by $p\left(\frac{1}{2}\right)$

$= \left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 1$

$= \frac{1}{8} + 3 \times \frac{1}{4} + 3 \times \frac{1}{2} + 1$

$= \frac{1 + 6 + 12 + 8}{8} = \frac{27}{8}$

(iii) x

Putting $x = 0$, we get

Using remainder theorem, when $p(x) = x^3 + 3x^2 + 3x + 1$ is divided by x, remainder is given by $p(0)$

$= (0)^3 + 3(0)^2 + 3(0) + 1$

$= 0 + 1 = 0$

(iv) $x + \pi$

Putting $x + \pi = 0$, we get, $x = -\pi$

Using remainder theorem, when $p(x) = x^3 + 3x^2 + 3x + 1$ is divided by $x + \pi$, remainder is given by $p(-\pi)$

$= (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1$

$= -\pi^3 + 3\pi^2 - 3\pi + 1$

(v) $5 + 2x$

Putting $5 + 2x = 0$, we get, $x = -\frac{5}{2}$

Using remainder theorem, when $p(x) = x^3 + 3x^2 + 3x + 1$ is divided by $5 + 2x$, remainder is given by $p\left(-\frac{5}{2}\right)$

$= \left(-\frac{5}{2}\right)^3 + 3\left(-\frac{5}{2}\right)^2 + 3\left(-\frac{5}{2}\right) + 1$

$= -\frac{125}{8} + 3 \times \frac{25}{4} - 3 \times \frac{5}{2} + 1$

$= \frac{-125 + 150 - 60 + 8}{8}$

$= -\frac{27}{8}$

Question 2:

Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

Answer 2:

Let $p(x) = x^3 - ax^2 + 6x - a$

Putting $x - a = 0$, we get, $x = a$

Using remainder theorem, when $p(x) = x^3 - ax^2 + 6x - a$ is divided by $x - a$, remainder is given by $p(a)$

$= (a)^3 - a(a)^2 + 6(a) - a$

$= a^3 - a^3 + 6a - a$

$= 5a$

Question 3:

Check whether $7 + 3x$ is a factor of $3x^3 + 7x$.

Answer 3:

Let $p(x) = 3x^3 + 7x$

Putting $7 + 3x = 0$, we get, $x = -\frac{7}{3}$

Using remainder theorem, when $p(x) = 3x^3 + 7x$ is divided by $7 + 3x$, remainder is given by $p\left(-\frac{7}{3}\right)$

$= 3\left(-\frac{7}{3}\right)^3 + 7\left(-\frac{7}{3}\right)$

$= -\frac{343}{27} - \frac{49}{3}$

$= \frac{-343 - 147}{9} = -\frac{490}{9}$

Since, remainder $p\left(-\frac{7}{3}\right) \neq 0$, hence, $7 + 3x$ is not a factor of $3x^3 + 7x$.

Exercise 2.4

Question 1:

Determine which of the following polynomials has $x + 1$ a factor:

(i) $x^3 + x^2 + x + 1$ **(ii)** $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$ **(iv)** $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

Answer 1:

(i) Let $p(x) = x^3 + x^2 + x + 1$

Putting $x + 1 = 0$, we get, $x = -1$

Using remainder theorem, when $p(x) = x^3 + x^2 + x + 1$ is divided by $x + 1$, remainder is given by $p(-1)$

$= (-1)^3 + (-1)^2 + (-1) + 1 = -1 + 1 - 1 + 1 = 0$

Since, remainder $p(-1) = 0$, hence $x + 1$ is a factor of $x^3 + x^2 + x + 1$.

(ii) Let $p(x) = x^4 + x^3 + x^2 + x + 1$

Putting $x + 1 = 0$, we get, $x = -1$

Using remainder theorem, when $p(x) = x^4 + x^3 + x^2 + x + 1$ is divided by $x + 1$, remainder is given by $p(-1)$

$= (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1 = 1 - 1 + 1 - 1 + 1 = 1$

Since, remainder $p(-1) \neq 0$, hence $x + 1$ is not a factor of $x^4 + x^3 + x^2 + x + 1$.

(iii) Let $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

Putting $x + 1 = 0$, we get, $x = -1$

Using remainder theorem, when $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$ is divided by $x + 1$, remainder is given by $p(-1)$

$= (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1 = 1 - 3 + 3 - 1 + 1 = 1$

Since, remainder $p(-1) \neq 0$, hence $x + 1$ is not a factor of $x^4 + 3x^3 + 3x^2 + x + 1$.

(iv) Let $p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

Putting $x + 1 = 0$, we get, $x = -1$

Using remainder theorem, when $p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$ is divided by $x + 1$, remainder is given by $p(-1)$

$= (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2} = -1 - 1 + 2 + \sqrt{2} + \sqrt{2} = 2\sqrt{2}$

Since, remainder $p(-1) \neq 0$, hence $x + 1$ is not a factor of $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$.

Question 2:

Use the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x) = 2x^3 + x^2 - 2x - 1,\ g(x) = x + 1$ **(ii)** $p(x) = x^3 + 3x^2 + 3x + 1,\ g(x) = x + 2$

(iii) $p(x) = x^3 - 4x^2 + x + 6,\ g(x) = x - 3$

Answer 2:

(i) $p(x) = 2x^3 + x^2 - 2x - 1$ and $g(x) = x + 1$

Putting $x + 1 = 0$, we get, $x = -1$

Using remainder theorem, when $p(x) = 2x^3 + x^2 - 2x - 1$ is divided by $g(x) = x + 1$, remainder is given by $p(-1)$

(ii) $x^3 - 3x^2 - 9x - 5$

Let $p(x) = x^3 - 3x^2 - 9x - 5$

Putting $x = 1$, we get

$p(1) = (1)^3 - 3(1)^2 - 9(1) - 5 = 1 - 3 - 9 - 5 = -16 \neq 0$

Putting $x = -1$, we get

$p(-1) = (-1)^3 - 3(-1)^2 - 9(-1) - 5 = -1 - 3 + 9 - 5 = 0$

$\Rightarrow x + 1$ is a factor of $p(x)$.

$$
\begin{array}{r|l}
 & x^2 - 4x - 5 \\
\hline
x + 1 & x^3 - 3x^2 - 9x - 5 \\
 & x^3 + x^2 \\
 & - \quad - \\
\hline
 & \quad -4x^2 - 9x - 5 \\
 & \quad -4x^2 - 4x \\
 & \quad + \qquad + \\
\hline
 & \qquad\qquad -5x - 5 \\
 & \qquad\qquad -5x - 5 \\
 & \qquad\qquad + \qquad + \\
\hline
 & \qquad\qquad 0
\end{array}
$$

So, $p(x) = (x + 1)(x^2 - 4x - 5)$

$= (x + 1)(x^2 - 4x - 5)$

$= (x + 1)[x^2 - (5 - 1)x - 5]$

$= (x + 1)[x^2 - 5x + x - 5]$

$= (x + 1)[x(x - 5) + 1(x - 5)]$

$= (x + 1)(x - 5)(x + 1)$

(iii) $x^3 + 13x^2 + 32x + 20$

Let $p(x) = x^3 + 13x^2 + 32x + 20$

Putting $x = -1$, we get

$p(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20 = -1 + 13 - 32 + 20 = 0$

$\Rightarrow x + 1$ is a factor of $p(x)$.

$$
\begin{array}{r|l}
 & x^2 + 12x + 20 \\
\hline
x + 1 & x^3 + 13x^2 + 32x + 20 \\
 & x^3 + x^2 \\
 & - \quad - \\
\hline
 & \quad 12x^2 + 32x + 20 \\
 & \quad 12x^2 + 12x \\
 & \quad - \qquad - \\
\hline
 & \qquad\qquad 20x + 20 \\
 & \qquad\qquad 20x + 20 \\
 & \qquad\qquad - \qquad - \\
\hline
 & \qquad\qquad 0
\end{array}
$$

So, $p(x) = (x + 1)(x^2 + 12x + 20)$

$= (x + 1)(x^2 + 12x + 20)$

$= (x + 1)[x^2 + (10 + 2)x + 20]$

$= (x + 1)[x^2 + 10x + 2x + 20] = (x + 1)[x(x + 10) + 2(x + 10)]$

$= (x + 1)(x + 10)(x + 2)$

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Exercise 2.5

Question 1:

Use suitable identities to find the following products:

(i) $(x+4)(x+10)$ **(ii)** $(x+8)(x-10)$ **(iii)** $(3x+4)(3x-5)$

(iv) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$ **(v)** $(3-2x)(3+2x)$

Answer 1:

(i) $(x+4)(x+10)$

$= x^2+(4+10)x+4\times 10$ $[\because (x+a)(x+b)=x^2+(a+b)x+ab]$

$= x^2+14x+40$

(ii) $(x+8)(x-10)$

$= x^2+(8-10)x+8\times(-10)$ $[\because (x+a)(x+b)=x^2+(a+b)x+ab]$

$= x^2-2x-80$

(iii) $(3x+4)(3x-5)$

$= (3x)^2+(4-5)3x+4\times(-5)$ $[\because (x+a)(x+b)=x^2+(a+b)x+ab]$

$= 9x^2-3x-20$

(iv) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

$= (y^2)^2-\left(\frac{3}{2}\right)^2$ $[\because (a+b)(a-b)=a^2-b^2]$

$= y^4-\frac{9}{4}$

(v) $(3-2x)(3+2x)$

$= (3)^2-(2x)^2$ $[\because (a+b)(a-b)=a^2-b^2]$

$= 9-4x^2$

Question 2:

Evaluate the following products without multiplying directly:

(i) 103×107 **(ii)** 95×96 **(iii)** 104×96

Answer 2:

(i) 103×107

$= (100+3)(100+7)$

$= (100)^2+(3+7)100+3\times 7$ $[\because (x+a)(x+b)=x^2+(a+b)x+ab]$

$= 10000+1000+21=11021$

(ii) 95×96

$(100-5)(100-4)$

$= (100)^2+(-5-4)100+(-5)\times(-4)$ $[\because (x+a)(x+b)=x^2+(a+b)x+ab]$

$= 10000-900+20=9120$

(iii) 104×96

$(100+4)(100-4)=(100)^2-(4)^2$ $[\because (a+b)(a-b)=a^2-b^2]$

$= 10000-16=9984$

Question 3:

Factorise the following using appropriate identities:

(i) $9x^2+6xy+y^2$ **(ii)** $4y^2-4y+1$ **(iii)** $x^2-\frac{y^2}{100}$

Question 5:

Factorise:

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

Answer 5:

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2 \times (2x) \times 3y + 2 \times 3y \times (-4z) + 2 \times (-4z) \times (2x)$

$= (2x + 3y - 4z)^2$

$[\because a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2]$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

$= (\sqrt{2}x)^2 + (-y)^2 + (-2\sqrt{2}z)^2 + 2(\sqrt{2}x)(-y) + 2(-y)(-2\sqrt{2}z) + 2(-2\sqrt{2}z)(\sqrt{2}x)$

$= (\sqrt{2}x - y - 2\sqrt{2}z)^2$ $[\because a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2]$

Question 6:

Write the following cubes in expanded form:

(i) $(2x + 1)^3$ **(ii)** $(2a - 3b)^3$ **(iii)** $\left[\frac{3}{2}x + 1\right]^3$ **(iv)** $\left[x - \frac{2}{3}y\right]^3$

Answer 6:

(i) $(2x + 1)^3$

$= (2x)^3 + 1^3 + 3(2x)^2(1) + 3(2x)(1)^2$

$[\because (a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2]$

$= 8x^3 + 1 + 12x^2 + 6x$

(ii) $(2a - 3b)^3$

$= (2a)^3 + (-3b)^3 + 3(2a)^2(-3b) + 3(2a)(-3b)^2$

$[\because (a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2]$

$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$

(iii) $\left[\frac{3}{2}x + 1\right]^3$

$= \left(\frac{3}{2}x\right)^3 + 1^3 + 3\left(\frac{3}{2}x\right)^2(1) + 3\left(\frac{3}{2}x\right)(1)^2$

$[\because (a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2]$

$= \frac{27}{8}x^3 + 1 + \frac{27}{4}x^2 + \frac{9}{2}x$

(iv) $\left[x - \frac{2}{3}y\right]^3$

$= (x)^3 + \left(-\frac{2}{3}y\right)^3 + 3(x)^2\left(-\frac{2}{3}y\right) + 3(x)\left(-\frac{2}{3}y\right)^2$

$[\because (a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2]$

$= x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$

Question 7:

Evaluate the following using suitable identities:

(i) $(99)^3$ **(ii)** $(102)^3$ **(iii)** $(998)^3$

Answer 3:

(i) $9x^2 + 6xy + y^2$
$= (3x)^2 + 2 \times 3x \times y + y^2$
$= (3x + y)^2$ $\quad [\because a^2 + 2ab + b^2 = (a + b)^2]$

(ii) $4y^2 - 4y + 1$
$= (2y)^2 - 2 \times 2y \times 1 + 1^2$
$= (2y - 1)^2$ $\quad [\because a^2 - 2ab + b^2 = (a - b)^2]$

(iii) $x^2 - \frac{y^2}{100}$
$= x^2 - \left(\frac{y}{10}\right)^2$
$= (x + \frac{y}{10})(x - \frac{y}{10})$ $\quad [\because a^2 - b^2 = (a + b)(a - b)]$

Question 4:

Expand each of the following, using suitable identities:

(i) $(x + 2y + 4z)^2$ **(ii)** $(2x - y + z)^2$ **(iii)** $(-2x + 3y + 2z)^2$
(iv) $(3a - 7b - c)^2$ **(v)** $(-2x + 5y - 3z)^2$ **(vi)** $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$

Answer 4:

(i) $(x + 2y + 4z)^2$
$= x^2 + (2y)^2 + (4z)^2 + 2 \times (x) \times (2y) + 2 \times (2y) \times (4z) + 2 \times (4z) \times (x)$
$[\because (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca]$
$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 4zx$

(ii) $(2x - y + z)^2$
$= (2x)^2 + (-y)^2 + (z)^2 + 2 \times (2x) \times (-y) + 2 \times (-y) \times (z) + 2 \times (z) \times (2x)$
$[\because (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca]$
$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4zx$

(iii) $(-2x + 3y + 2z)^2$
$= (-2x)^2 + (3y)^2 + (2z)^2 + 2 \times (-2x) \times (3y) + 2 \times (3y) \times (2z) + 2 \times (2z) \times (-2x)$
$[\because (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca]$
$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8zx$

(iv) $(3a - 7b - c)^2$
$= (3a)^2 + (-7b)^2 + (-c)^2 + 2 \times (3a) \times (-7b) + 2 \times (-7b) \times (-c) + 2 \times (-c) \times (3a)$
$[\because (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca]$
$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ca$

(v) $(-2x + 5y - 3z)^2$
$= (-2x)^2 + (5y)^2 + (-3z)^2 + 2 \times (-2x) \times 5y + 2 \times 5y \times (-3z) + 2 \times (-3z) \times (-2x)$
$[\because (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca]$
$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx$

(vi) $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$
$= \left(\frac{1}{4}a\right)^2 + \left(-\frac{1}{2}b\right)^2 + 1^2 + 2 \times \left(\frac{1}{4}a\right) \times \left(-\frac{1}{2}b\right) + 2 \times \left(-\frac{1}{2}b\right) \times 1 + 2 \times 1 \times \left(\frac{1}{4}a\right)$
$[\because (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca]$
$= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a$

Answer 7:

(i) $(99)^3$

$(100-1)^3$

$= (100)^3 + (-1)^3 + 3(100)^2(-1) + 3(100)(-1)^2$

$[\because (a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2]$

$= 1000000 - 1 - 30000 + 300 = 970299$

(ii) $(102)^3$

$(100+2)^3$

$= (100)^3 + (2)^3 + 3(100)^2(2) + 3(100)(2)^2$

$[\because (a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2]$

$= 1000000 + 8 + 60000 + 1200 = 1061208$

(iii) $(998)^3$

$(1000-2)^3$

$= (1000)^3 + (-2)^3 + 3(1000)^2(-2) + 3(1000)(-2)^2$

$[\because (a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2]$

$= 1000000000 - 8 - 6000000 + 12000 = 994011992$

Question 8:

Factorise each of the following:

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii) $27 - 125a^3 - 135a + 225a^2$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Answer 8:

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

$= (2a)^3 + (b)^3 + 3(2a)^2(b) + 3(2a)(b)^2$

$= (2a+b)^3$ $[\because (a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2]$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

$= (2a)^3 + (-b)^3 + 3(2a)^2(-b) + 3(2a)(-b)^2$

$= (2a-b)^3$ $[\because (a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2]$

(iii) $27 - 125a^3 - 135a + 225a^2$

$= (3)^3 + (-5a)^3 + 3(3)^2(-5a) + 3(3)(-5a)^2$

$= (3-5a)^3$ $[\because (a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2]$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

$= (4a)^3 + (-3b)^3 + 3(4a)^2(-3b) + 3(4a)(-3b)^2$

$= (4a-3b)^3$ $[\because (a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2]$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

$= (3p)^3 + \left(-\frac{1}{6}\right)^3 + 3(3p)^2\left(-\frac{1}{6}\right) + 3(3p)\left(-\frac{1}{6}\right)^2$

$= \left(3p - \frac{1}{6}\right)^3$ $[\because (a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2]$

Question 9:

Verify:

(i) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

(ii) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

Question 13:

If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$.

Answer 13:

We know that $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$
If $x + y + z = 0$, then $x^3 + y^3 + z^3 - 3xyz = (0)(x^2 + y^2 + z^2 - xy - yz - zx)$
$\Rightarrow x^3 + y^3 + z^3 - 3xyz = 0 \Rightarrow x^3 + y^3 + z^3 = 3xyz$

Question 14:

Without actually calculating the cubes, find the value of each of the following:
(i) $(-12)^3 + (7)^3 + (5)^3$ (ii) $(28)^3 + (-15)^3 + (-13)^3$

Answer 14:

(i) $(-12)^3 + (7)^3 + (5)^3$
Let, $a = -12,\ b = 7,\ c = 5$
So, $a + b + c = -12 + 7 + 5 = 0$
We know that if $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$
Therefore, $(-12)^3 + (7)^3 + (5)^3 = 3(-12)(7)(5) = -1260$
(ii) $(28)^3 + (-15)^3 + (-13)^3$
Let, $a = 28,\ b = -15,\ c = -13$, so, $a + b + c = 28 - 15 - 13 = 0$
We know that if $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$
Therefore, $(28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13) = 16380$

Question 15:

Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:
(i) Area: $25a^2 - 35a + 12$ (ii) Area: $35y^2 + 13y - 12$

Answer 15:

(i) Area: $25a^2 - 35a + 12$
$= 25a^2 - 20a - 15a + 12 = 5a(5a - 4) - 3(5a - 4) = (5a - 4)(5a - 3)$
Therefore, length = $(5a - 3)$ and breadth = $(5a - 4)$ [as $(5a - 3) > (5a - 4)$]
(ii) Area: $35y^2 + 13y - 12$
$= 35y^2 + 28y - 15y - 12 = 7y(5y + 4) - 3(5y + 4) = (5y + 4)(7y - 3)$
Therefore, length = $(5y + 4)$ and breadth = $(7y - 3)$

Question 16:

What are the possible expressions for the dimensions of the cuboids whose volumes are given below?
(i) Volume = $3x^2 - 12x$ (ii) Volume = $12ky^2 + 8ky - 20k$

Answer 16:

(i) Volume = $3x^2 - 12x$
$= 3x(x - 4) = (3)(x)(x - 4)$
Hence, the possible dimensions of the cuboids are 3, x and $x - 4$.
(ii) Volume = $12ky^2 + 8ky - 20k$
$= 4k(3y^2 + 2y - 5) = 4k(3y^2 + 5y - 3y - 5) = 4k[y(3y + 5) - 1(3y + 5)]$
$= (4k)(3y + 5)(y - 1)$
Hence, the possible dimensions of the cuboids are $4k$, $(3y + 5)$ and $(y - 1)$.

Answer 9:

(i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

RHS

$= (x + y)(x^2 - xy + y^2) = x(x^2 - xy + y^2) + y(x^2 - xy + y^2)$

$= x^3 - x^2y + xy^2 + yx^2 - xy^2 + y^3 = x^3 + y^3 = LHS$

(ii) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

RHS

$= (x - y)(x^2 + xy + y^2) = x(x^2 + xy + y^2) - y(x^2 + xy + y^2)$

$= x^3 + x^2y + xy^2 - yx^2 - xy^2 - y^3 = x^3 - y^3 = LHS$

Question 10:

Factorise each of the following:

(i) $27y^3 + 125z^3$ **(ii)** $64m^3 - 343n^3$

Answer 10:

(i) $27y^3 + 125z^3$

$= (3y)^3 + (5z)^3$

$= (3y + 5z)[(3y)^2 - (3y)(5z) + (5z)^2]$ $[\because x^3 + y^3 = (x + y)(x^2 - xy + y^2)]$

$= (3y + 5z)(9y^2 - 15yz + 25z^2)$

(ii) $64m^3 - 343n^3$

$= (4m)^3 - (3n)^3$

$= (4m - 3n)[(4m)^2 + (4m)(3n) + (3n)^2]$ $[\because x^3 - y^3 = (x - y)(x^2 + xy + y^2)]$

$= (4m - 3n)(16m^2 + 12mn + 9n^2)$

Question 11:

Factorise: $27x^3 + y^3 + z^3 - 9xyz$

Answer 11:

$27x^3 + y^3 + z^3 - 9xyz = (3x)^3 + y^3 + z^3 - 3(3x)yz$

$= (3x + y + z)[(3x)^2 + y^2 + z^2 - (3x)y - yz - z(3x)]$

$[\because a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)]$

$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3zx)$

Question 12:

Verify that: $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

Answer 12:

$RHS = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

$= \frac{1}{2}(x + y + z)[x^2 + y^2 - 2xy + y^2 + z^2 - 2yz + z^2 + x^2 - 2zx][\because (a - b)^2 = a^2 + b^2 - 2ab]$

$= \frac{1}{2}(x + y + z)(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx)$

$= \frac{1}{2} \times 2(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= x^3 + y^3 + z^3 - 3xyz = LHS$

$[\because (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc]$